

The Complexity of Manipulative Attacks in Nearly Single-Peaked Electorates

[Extended Abstract]

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ABSTRACT

Many electoral bribery, control, and manipulation problems (which we will refer to in general as “manipulative actions” problems) are NP-hard in the general case. It has recently been noted that many of these problems fall into polynomial time if the electorate is single-peaked (i.e., is polarized along some axis/issue). However, real-world electorates are not truly single-peaked. There are usually some mavericks, and so real-world electorates tend to merely be nearly single-peaked. This paper studies the complexity of manipulative-action algorithms for elections over nearly single-peaked electorates, for various notions of nearness and various election systems. We provide instances where even one maverick jumps the manipulative-action complexity up to NP-hardness, but we also provide many instances where a reasonable number of mavericks can be tolerated without increasing the manipulative-action complexity.

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1. INTRODUCTION

Elections are a model of collective decision-making so central in human and multiagent-systems contexts—ranging

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from planning to collaborative filtering to reducing web spam—that it is natural to want to get a handle on the computational difficulty of finding whether manipulative actions can obtain a given outcome (see the survey [21]). A recent line of work started by Walsh [35, 19, 4] has looked at the extent to which NP-hardness results for the complexity of manipulative actions (bribery, control, and manipulation) may evaporate when one focuses on electorates that are (unidimensional) single-peaked, a central social-science model of electoral behavior. That model basically views society as polarized along some (perhaps hidden) issue or axis. However, real-world elections are unlikely to be perfectly single-peaked. Rather, they are merely very close to being single-peaked, a notion that was recently raised in a computational context by Conitzer [6] and Escoffier et al. [15]. There will almost always be a few mavericks, who vote based on some reason having nothing to do with the societal axis. For example, in recent US presidential primary and final elections, there was much discussion of whether some voters would vote not based on the political positioning of the candidates but rather based on the candidates’ religion, race, or gender. In this paper, we most centrally study whether the evaporation of complexity results that often holds for single-peaked electorates will also occur in nearly single-peaked electorates. We prove that often the answer is yes, and sometimes the answer is no. We defer to Section 6 our discussion of previous and related work.

Among the contributions of our paper are the following.

- Most centrally, we show that in many control and bribery settings, a reasonable number of mavericks (voters whose votes are not consistent with the societal axis) can be handled. In such cases, the “complexity-shield evaporation” results of the earlier work can now be declared free from the worry that the results might hold only for perfect single-peakedness.
- We give settings, for example 3-candidate Borda and 3-candidate veto, in which even one maverick raises the (constructive coalition weighted) manipulation complexity from P to NP-hardness.
- For all scoring systems of the form $(\alpha_1, \alpha_2, \alpha_3)$, $\alpha_2 \neq \alpha_3$, we provide a dichotomy theorem determining when the (constructive coalition weighted) manipulation problem is in P and when it is NP-complete, for so-called single-caved societies.

- We show cases where the price of mavericity is paid in nondeterminism—cases where, for each k , we prove the control problem for societies with $\mathcal{O}(\log^k n)$ mavericks to be in complexity class β_k , the k th level of the limited nondeterminism hierarchy of Kintala and Fisher [30].

This paper touches on bribery, control, and manipulation, discusses various election systems and notions of nearness to single-peaked, and gives both polynomial-time attack results and NP-hardness results. It thus is not surprising that the proofs vary broadly in their techniques and approaches; we have no single approach that covers this entire range of cases. Due to space limitations we omit almost all proofs from this paper, but they are available through our technical report [22].

2. PRELIMINARIES

In this section we give intuitive descriptions of the problems that we study. More detailed coverage, and discussion of the motivations and limitations of the models, can be found in the various bibliography entries, including, for example, Faliszewski et al. [18, 21]. We have also included formal definitions in the appendix.

Elections An election $E = (C, V)$ consists of a finite candidate set C and a finite collection V of votes over the candidates. V is a list of entries, one per voter, with each entry containing a linear (i.e., tie-free total) ordering of the candidates (except for approval elections where each vote is a $\|C\|$ -long 0-1 vector denoting disapproval/approval of each candidate).¹ In plurality elections, whichever candidate gets the most top-of-the-preference-order votes wins. Each vector $(\alpha_1, \dots, \alpha_k)$, $\alpha_i \in \mathbb{N}$, $\alpha_1 \geq \dots \geq \alpha_k \geq 0$, defines a k -candidate scoring protocol election, in which each voter’s i th favorite candidate gets α_i points, and whichever candidate gets the most points wins. k -candidate veto is defined by the

vector $(\overbrace{1, \dots, 1}^{k-1}, 0)$, and k -candidate Borda is defined by the vector $(k-1, k-2, \dots, 0)$. In approval elections, whichever candidate is approved of by the most voters wins. In all the systems just mentioned, if candidates tie for the highest number of points, those tying for highest are all considered winners. In Condorcet elections, a candidate wins if he or she strictly beats every other candidate in pairwise head-on-head votes.

Attacks Each of the election problems is defined based on an election and some additional parameters. In constructive coalition weighted manipulation (CCWM), the input is a set of nonmanipulative voters (having weights and preferences over the candidates), a list of the weights of the manipulative voters, and which candidate p the manipulators wish to be a winner. (Input) instances are in the set exactly if there is a set of votes the manipulators can cast to make p a winner (under the given election system). In “bribery,” all voters have preferences, and our input is an election, a candidate p , and a bound K on how many voters

¹Throughout this paper V , though input as a list (one ballot per voter), typically functions as a multiset. When dealing with voter sets, we use terms such as set/subset to mean multiset/submultiset (although we will typically just say collection), and we use set-bracket notation to, for that case, mean multisets, e.g., if v is a preference order, w is a preference order, $V = \{v, v\}$, and $W = \{v, w\}$, then $V \cup W = \{v, v, v, w\}$.

can be bribed. Instances are in the set if there is a way of changing the preferences of at most K voters that makes p a winner. (We will briefly mention a number of variations of bribery. “Weighted” means the voters have weights, “\$” means voters have individual prices (and K becomes a bound on the amount that can be spent seeking to make p win). For approval voting, “negative” bribery means a bribe cannot change someone from disapproving of p to approving of p , and “strongnegative” bribery means every bribed person must end up disapproving of p . In negative bribery, one can only help p in subtle, indirect ways. For plurality, the negative notion is similar, see Faliszewski et al. [17] or our appendix.) In control (of the four types we will discuss), the input is an election, the candidate p one wants to be a winner, and a parameter K limiting how many actors one can influence in the designated way. Our four types of control will be adding voters (CCAV), deleting voters (CCDV), adding candidates (CCAC), and deleting candidates (CCDC). Each of those four problems is defined as the collection of inputs on which using at most K actions of the designated type (e.g., adding at most K voters) suffices to make p a winner. For CCAV an additional part of the input is a pool of potential additional voters (and their preferences over the candidates). For CCAC an additional part of the input is a pool of potential additional candidates, and all voters have preferences over the set of all initial and potential-additional candidates. (Some early papers on control focused on making p be the one and only winner, but we follow the more recent approach of focusing on making p become a winner. The older results for the former case that we cite here are known in the literature, or were verified for this paper by us, to also hold for the latter case.) Control loosely models such real-world activities as get-out-the-vote drives, targeted advertising, and voter suppression.

Each of our algorithms for manipulation, bribery, and control not only gives a *yes/no* answer for the decision variant of the problem, but also can be made to produce a successful manipulative action if the answer is *yes*.

Having algorithms for such tasks as bribery and control isn’t inherently a “bad” or unethical thing. For example, bribery and control algorithms are valuable tools for actors (party chairs, campaign managers, etc.) who are trying to most effectively use their resources.

(Nearly) Single-peakedness A collection V of votes (cast as linear orders) is said to be single-peaked exactly if there is a linear order L over the candidate set such that for each triple of candidates, c_1, c_2, c_3 , it holds that if $c_1 L c_2 L c_3 \vee c_3 L c_2 L c_1$, then $(\forall v \in V)[c_1 P_v c_2 \implies c_2 P_v c_3]$, where $a P_v b$ means that voter v prefers a to b . This notion was first created by Black more than half a century ago, and is one of the most important concepts in political science. Loosely put, the notion is motivated as follows. Imagine that on some issue, for example what the tax rate should be for the richest Americans, each person has a utility curve that on a (perhaps empty) initial part is nondecreasing and then on the (perhaps empty) rest is nonincreasing. Suppose the candidates are spread along the tax-rate axis as to their positions, with no two on top of each other. The set of preferences that can be supported among them by curves of the mentioned sort on which there are no ties among candidates in utility are precisely the single-peaked vote ensembles. Note that different voters can have different peaks/plateaus and different curves, e.g., if both Alice and

Bob think 40 percent is the ideal top tax rate, it is completely legal for Alice to prefer 30 percent to 50 percent and Bob to prefer 50 percent to 30 percent. There is extensive political science literature on single-peaked voting’s naturalness, ranging from conceptual discussions to empirical studies of actual US political elections (with few candidates) showing that most voters are single-peaked with respect to left-right political spectrum, and has been described as “*the canonical setting for models of political institutions*” [25]. If in the definition of single-peaked one replaces the final “forall” with this one, $(\forall v \in V)[c_2 P_v c_1 \implies c_3 P_v c_2]$, one defines the closely related notion of single-caved preferences, which we will also study. For approval ballots, a vote set V is said to be single-peaked if there is a linear order L such that for each voter v , all candidates that v approves of (if any) form an adjacent block in L .

In all our manipulative action problems about single-peaked and nearly single-peaked societies, we will follow Walsh’s model, which is that the societal order, L , is part of the input. (See the earlier papers for extensive discussion of why this is a reasonable model.)

In this paper, we will primarily focus on elections whose voters are “nearly” single-peaked, under the following notions of nearness. Our “maverick” notions apply to both voting by approval ballots and voting by linear orders; our other notions are specific to voting by linear orders. We will say an election is over a k -maverick-SP society (equivalently, a k -maverick-SP electorate) if all but k of the voters are consistent with (in the sense of single-peakedness; this does not mean identical to) the societal order L . That is, we allow up to k mavericks. We will speak of $f(\cdot)$ -maverick-SP societies when this usage and the type of f ’s argument(s) is clear from context (f ’s argument(s) will typically be the size of the election instance or some parameters of the election, e.g., the number of candidates or the number of voters).

Also, we will prove a number of results that state that “PROBLEM for ELECTION-SYSTEM over log-maverick-SP societies is in P”; this is a shorthand for the claim that for each function f (that is computable in time polynomial in the size of the input—which is roughly $\|V\| \|C\| \log \|C\|$ for the election (C, V) itself plus whatever space is taken by other parameters—and to avoid possible technical problems, we should assume f is nondecreasing) whose *value* is $\mathcal{O}(\log(\text{ProblemInputSize}))$, it holds that “PROBLEM for ELECTION-SYSTEM over f -maverick-SP societies is in P,” where the argument to f is the input size of the problem. An election is over a (k, k') -swoon-SP society if each voter has the property that if one removes the voter’s k favorite and k' least favorite candidates from the voter’s preference order, the resulting order is consistent with societal order L after removing those same candidates from L . We will use swoon-SP as a shorthand for $(1, 0)$ -swoon-SP, as we will not study other swoon values in this paper. In swoon-SP, each person may have as her or his favorite some candidate chosen due to some personal passion (such as hairstyle or religion), but all the rest of that person’s vote must be consistent with the societal polarization. An election is over a Dodgson $_k$ -SP society if for each voter some at-most- k sequential exchanges of adjacent candidates in his or her order make the vote consistent with the societal order L . An election is over a PerceptionFlip $_k$ -SP society if, for each voter, there is some series of at most k sequential exchanges of adjacent candidates *in the societal order* L after which the

voter’s vote is consistent with L . This models each voter being consistent with that voter’s humanly blurred view of the societal order.

Some model details follow. For control by adding voters for maverick-SP societies, the total number of mavericks in the initial voter set and the pool of potential additional voters is what the maverick bound limits. For manipulation, nonmanipulators as well as manipulators can be mavericks, and we bound the total number of mavericks. For bribery involving $f(\cdot)$ -maverick-SP societies, we will consider both the “standard” model and the “marked” model. In the standard model, the $f(\text{ProblemInputSize})$ limit on the number of mavericks must hold both for the input and for the voter set after the bribing is done; anyone may be bribed and bribes can create mavericks and can make mavericks become nonmavericks. In the marked model, each voter has a flag saying whether or not he or she can cast a maverick vote (we will call that being “maverick-enabled”). The at most $f(\text{ProblemInputSize})$ voters with the maverick-enabled flag may (subject to the other constraints of the bribery problem such as total number of bribes) be bribed in any way, and so may legally cross in either direction between consistency and inconsistency with the societal ordering. All non-maverick-enabled voters must be consistent with societal order L both before and after the bribing, although they too can be bribed (again, subject to the problem’s other constraints such as total number of bribes).

For single-peaked electorates, “median voting” (in which the candidate wins who on the societal axis is preferred by the “median voter”) is known to be strategy-proof, i.e., a voter never benefits from misrepresenting his or her preferences. It might seem tempting to conclude from that that all elections on single-peaked societies “should” use median voting, and that we thus need not discuss single-peaked (or perhaps even nearly single-peaked) elections with respect to other voting systems, such as plurality, veto, etc. But that temptation should be resisted. First, median voting’s strategy-proofness regards manipulation, not control or bribery. Second, even in real-world political elections broadly viewed as being (nearly) single-peaked, it simply is not the case that median voting is used. People, for whatever reasons of history and comfort, use such systems as plurality, approval, and so on for such elections. And so algorithms for those systems are worth studying. Third, for manipulation of nearly single-peaked electorates, strategy-proofness does not even hold. And although for them indeed only the mavericks can have an incentive to lie, that doesn’t mean that the outcome won’t be utterly distorted even by a single maverick. There are arbitrarily large electorates, having just one maverick, where that maverick can change the winner from being the median one to instead being a candidate on the outer extreme of the societal order.

3. MANIPULATION

This paper’s sections on control and bribery focus on, and provide many examples of, settings where not just the single-peaked case but even the nearly single-peaked cases have polynomial-time algorithms. Regarding manipulation, the results are more sharply varied.

We show that NP-hardness holds for a rich class of scoring protocols, in the presence of even one maverick. (When $\alpha_2 = \alpha_3$ the system is either equivalent to plurality or is a trivial system where everyone always is a winner. These

cases are easily seen to be in P.) Recall from Section 2 the meaning of “ $(\alpha_1, \alpha_2, \alpha_3)$ elections,” namely, scoring protocol elections using the vector $(\alpha_1, \alpha_2, \alpha_3)$.

THEOREM 1. *For each $\alpha_1 \geq \alpha_2 > \alpha_3$, CCWM for $(\alpha_1, \alpha_2, \alpha_3)$ elections over 1-maverick-SP societies is NP-complete.*

We point out that this theorem is of the same form as that for the general case (see [7, 27, 33]). However, the proofs for the general case do not work in our case, since those proofs construct elections with at least two mavericks.

In the general case (i.e., no single-peakedness is required), the above cases also are NP-complete [7, 27, 33], so allowing a one-maverick single-peaked society is jumping us up to the same level of complexity that holds in the general case here. In contrast, for SP societies (without mavericks), 3-candidate CCWM is NP-complete when $(\alpha_1 - \alpha_3) > 2(\alpha_2 - \alpha_3) > 0$ and is in P otherwise [23]. So, in particular, 3-candidate veto and 3-candidate Borda elections are in P for the SP (single-peaked) case, but are already NP-complete for SP with one maverick allowed.

Does allowing one maverick always raise the CCWM complexity? No, as the following theorem shows. (The $k = 0$ case follows from Faliszewski et al. [23].)

THEOREM 2. *For each $k \geq 0$ and $m \geq k + 3$, CCWM for m -candidate veto elections over k -maverick-SP societies is in P.*

In contrast, all of Theorem 2’s cases are well-known to be NP-complete in the general case [7]. Still, the contrast is a bit fragile. For example, although the above theorem shows that CCWM for $(1, 1, 1, 1, 0)$ elections over 2-maverick-SP societies is in P, we prove below that CCWM for $(1, 1, 1, 0)$ elections over 2-maverick-SP societies is NP-complete. Note also that this theorem gives an example where 4-candidate veto elections are NP-complete but 5-candidate veto elections are in P, in contrast with the behavior that one often expects regarding NP-completeness and parameters, namely, one might expect that increasing the number of candidates wouldn’t lower the complexity. (However, see Faliszewski et al. [23] for another example of this unusual behavior.)

THEOREM 3. *For each $k \geq 0$ and $m \geq 3$ such that $m \leq k + 2$, CCWM for m -candidate veto elections over k -maverick-SP societies is NP-complete.*

Theorems 2 and 3 also show that for any number of mavericks, there exists a voting system such that CCWM is easy for up to that number of mavericks, and hard for more mavericks.

COROLLARY 4. *Let $k \geq 0$. For all $\ell \geq 0$, CCWM for $k + 3$ -candidate veto elections over ℓ -maverick-SP societies is in P if $\ell \leq k$ and NP-complete otherwise.*

Let us now turn from the maverick notion of nearness to single-peakedness, and look at the “swoon” notion, in which, recall, each voter must be consistent with the societal ordering (minus the voter’s first-choice candidate) when one removes from the voter’s preference list the voter’s first-choice candidate. We will still mostly focus on the case of veto elections. For three candidates (see Observation 6) and four

candidates we have NP-completeness, and for five or more candidates we have membership in P.

THEOREM 5. *For each $m \geq 5$, CCWM for m -candidate veto elections in swoon-SP societies is in P. For $m \in \{3, 4\}$, this problem is NP-complete.*

OBSERVATION 6. *Every 3-candidate election is a swoon-SP election and a Dodgson₁-SP election and so all complexity results for 3-candidate elections in the general case also hold for swoon-SP elections and Dodgson₁-SP elections.*

Complexity results for general elections do not always hold for swoon-SP elections or for Dodgson₁-SP elections. For example, for $m \geq 5$, CCWM for m -veto elections is NP-complete in the general case, but in P for swoon-SP societies (Theorem 5) and Dodgson₁-SP societies (Theorem 7).

THEOREM 7. *For each $m \geq 5$, CCWM for m -candidate veto elections in Dodgson₁-SP societies is in P. For $m \in \{3, 4\}$, this problem is NP-complete.*

We conclude this section with a brief comment about single-caved electorates. (We remind the reader that single-caved is not a “nearness to SP” notion, but rather is in some sense a mirror-sibling.) For scoring vectors $(\alpha_1, \alpha_2, \alpha_3)$, the known CCWM dichotomy result for single-peaked electorates is that if $\alpha_1 - \alpha_3 > 2(\alpha_2 - \alpha_3)$ then the problem is NP-complete and otherwise the problem is in P. For single-caved, the opposite holds for each case that is not in P in the general case.

THEOREM 8. *For each $\alpha_1 \geq \alpha_2 > \alpha_3$, CCWM for $(\alpha_1, \alpha_2, \alpha_3)$ elections over single-caved societies is NP-complete if $(\alpha_1 - \alpha_3) \leq 2(\alpha_2 - \alpha_3)$ and otherwise is in P.*

4. CONTROL

The very first results of Faliszewski et al. [23] showing that NP-complete general-case control results can simplify to P results for single-peaked electorates were for constructive control by adding voters and for constructive control by deleting voters, for approval elections. We show that each of those results can be reestablished even in the presence of logarithmically many mavericks. (Indeed, we mention in passing that even if the attacker is allowed to simultaneously both add and delete voters—so-called “AV+DV” multimode control in the recent model that allows simultaneous attacks [16]—the complexity of planning an optimal attack still remains polynomial-time even with logarithmically many mavericks.)

THEOREM 9. *CCAV and CCDV for approval elections over log-maverick-SP societies are each in P. For CCAV, the complexity remains in P even for the case where no limit is imposed on the number of mavericks in the initial voter set, and the number of mavericks in the set of potential additional voters is logarithmically bounded (in the overall problem input size).²*

²By that last part, we mean precisely the definition—including its various restrictions on the complexity of the function—of the notion of log-maverick-SP, except with the limit being placed just on the number of mavericks in the additional voter set. Formally put, to avoid any possibility of ambiguity or confusion, we mean that for each

Although we will soon prove a more general result, we start by proving this result directly. We do so both as that will make the more general result clearer, and as P-time results are the core focus of this paper. Our proof involves the “demaverickification” of the society, in order to allow us to exploit the power of single-peakedness. By doing so, our proof establishes that there is a polynomial-time disjunctive truth-table reduction (see Ladner et al. [31] for the formal definition of \leq_{dt}^p , but one does not need to know that to follow our proof) to the single-peaked case. In this version of this paper, the proof of Theorem 9 is due to space not included, but it can be found in the on-line full technical-report version [22], and we assume that the interested reader will now read it (doing so is not required and probably not even a good idea on a first reading of this version, but such reading will make clearer the next paragraph, which briefly refers to the algorithm within that proof). Now, before we move on to other election systems, let us pause to wonder whether Theorem 9 is just the tip of an iceberg, and is in fact hiding some broader connection between number of mavericks and computational complexity theory. We won’t give this type of discussion for all, or even most, of our theorems. But it is worthwhile to, since this is our first control result, look here at what holds. And what holds is that Theorem 9 is indeed in some sense the tip of an iceberg. However, it is an iceberg whose tip is its most interesting part, since it gives the part that admits polynomial-time attacks.

Still, the rest of the iceberg brings out an interesting connection between maverick frequency and nondeterminism. Let us think again of the proof we just saw. It worked by sequentially generating each member of the powerset of a logarithmic-sized set (call it Q). And we did that, naturally enough, in polynomial time. However, note that we could also have done it with nondeterminism. We can nondeterministically guess for each member of Q whether or not it will be added (for CCAV) or deleted (for CCDV). And then after that nondeterministic guess, we for the CCAV case do the demaverickification presented in the above theorem’s proof and for the CCDV case do the deletable/nondeletable marking, and then we run the polynomial-time algorithms for the single-peaked approval-voting CCAV and the approval-voting CCDV (with deletable/nondeletable flag, and all mavericks—those not consistent with the societal ordering—being nondeletable) cases. It is easy to see that above proof argument works fine with the change to nondeterminism. Indeed, the reason Theorem 9 is about “P” is because sequentially handling $\mathcal{O}(\log(\text{ProblemInputSize}))$ nondeterministic bits can be done in polynomial time.

So, what underlies the above theorem are the following results that say that frequency of mavericks in one’s society exacts a price, in nondeterminism. (We here are proving just an upper bound, but we conjecture that the connection is quite tight—that commonality of wild voter behavior is very closely connected with nondeterminism.) To

function f (that is computable in time polynomial in the size of the input; and to avoid possible technical problems, we require that f be nondecreasing) whose value is $\mathcal{O}(\log(\text{ProblemInputSize}))$, it holds that “CCAV for approval elections over inputs for which (as is standard in our model regarding SP and nearly-SP cases, a societal ordering of single-peakedness is given a part of the input, and) at most $f(\text{ProblemInputSize})$ of the additional voters are inconsistent with the societal ordering is in P.”

state the results, we need to briefly introduce some notions from complexity theory. Complexity theorists often separate out the weighing of differing resources, putting bounds on each. The only such class we need here is the class of languages that can be accepted in time $t(n)$ on machines using $g(n)$ bits of nondeterminism, which is typically denoted $\text{Nondet-Time}[g(n), t(n)]$. The most widely known such classes are those of the limited nondeterminism hierarchy, known as the beta hierarchy, of Kintala and Fisher [30] (see also [8] and the survey [26]). β_k is the class of sets that can be accepted in polynomial time on machines that use $\mathcal{O}(\log^k n)$ bits of nondeterminism: $\beta_k = \{L \mid (\exists \text{ polynomial } t(n))(\exists g(n) \in \mathcal{O}(\log^k n))[L \in \text{Nondet-Time}[g(n), t(n)]]\}$, or for short, $\beta_k = \text{Nondet-Time}[\mathcal{O}(\log^k n), \text{poly}]$. Of course, $\beta_0 = \beta_1 = \text{P}$. We can now state our result, which says that frequency of mavericity is paid for in nondeterminism.

THEOREM 10. *CCAV and CCDV for approval elections over $f(\cdot)$ -maverick-SP societies are each in $\text{Nondet-Time}[f(\text{ProblemInputSize}), \text{poly}]$. For CCAV, the complexity remains in $\text{Nondet-Time}[f(\text{ProblemInputSize}), \text{poly}]$ even for the case where no limit is imposed on the number of mavericks in the initial voter set, and the number of mavericks in the set of potential additional voters is $f(\cdot)$ -bounded (in the overall problem input size).*

COROLLARY 11. *For each natural number k , CCAV and CCDV for approval elections over $\mathcal{O}(\log^k n)$ -maverick-SP societies are each in β_k . For CCAV, the complexity remains in β_k even for the case where no limit is imposed on the number of mavericks in the initial voter set, and the number of mavericks in the set of potential additional voters is $\mathcal{O}(\log^k n)$ -bounded (in the overall problem input size).*

Theorem 9 follows from the $k = 1$ case of this more general corollary. Now let us turn from our particularly detailed discussion of CCAV and CCDV for approval voting, and let us look at other election systems.

For Condorcet elections, both CCAV and CCDV are known to be NP-complete in the general case [3] but to be in P for single-peaked electorates [4]. For Condorcet, results analogous to those for approval under $f(\cdot)$ -maverick-SP societies hold.

THEOREM 12. *CCAV and CCDV for Condorcet elections over $f(\cdot)$ -maverick-SP societies are each in $\text{Nondet-Time}[f(\text{ProblemInputSize}), \text{poly}]$. For CCAV, the complexity remains in $\text{Nondet-Time}[f(\text{ProblemInputSize}), \text{poly}]$ even for the case where no limit is imposed on the number of mavericks in the initial voter set, and the number of mavericks in the set of potential additional voters is $f(\cdot)$ -bounded (in the overall problem input size).*

COROLLARY 13. *For each natural number k , CCAV and CCDV for Condorcet elections over $\mathcal{O}(\log^k n)$ -maverick-SP societies are each in β_k . For CCAV, the complexity remains in β_k even for the case where no limit is imposed on the number of mavericks in the initial voter set, and the number of mavericks in the set of potential additional voters is $\mathcal{O}(\log^k n)$ -bounded (in the overall problem input size).*

For plurality, the most important of systems, CCAV and CCDV are known to be in P for the general case [3], so there is no need to seek a maverick result there. However, CCAC and CCDC are both known to be NP-complete in the general case and in P in the single-peaked case. For both of those, a constant number of mavericks can be handled.

THEOREM 14. *For each k , CCAC and CCDC for plurality over k -maverick-SP societies are in P.*

As in the case of Theorem 9, the idea of the algorithm is to (polynomial-time disjunctively truth-table) reduce to the single-peaked case. However, here the mavericks require a more involved brute-force search and thus we can only handle a constant number of them.

Unfortunately, swooning cannot be handled at all (unless $P = NP$, of course). Also, allowing the number of mavericks to be some root of the input size cannot be handled.

THEOREM 15. *CCAC and CCDC for plurality elections over swoon-SP societies are NP-complete.*

THEOREM 16. *For each $\epsilon > 0$, CCAC and CCDC for plurality elections over I^ϵ -maverick-SP societies are NP-complete, where I denotes the input size.*

For the Dodgson and PerceptionFlip notions of nearness to single-peakedness, we can prove that allowing a constant number of adjacent-swaps (for each voter, separately, in the appropriate structure) still leaves the CCAC and CCDC problems in P. (We mention in passing that the following result holds not just for constructive control, but even for the concept, which we are not focusing on in this paper, of destructive control, in which one's goal is just to preclude a certain candidate from winning.)

THEOREM 17. *For each k , CCAC and CCDC for plurality elections are in P for Dodgson $_k$ -SP societies and for PerceptionFlip $_k$ -SP societies.*

Our algorithm exploits the local nature of adding/deleting candidates in single-peaked plurality elections. Formally, Faliszewski et al. [23] made the following observation.

LEMMA 18 (LEMMA 3.4 OF [23]). *Let (C, V) be an election where $C = \{c_1, \dots, c_m\}$ is a set of candidates, V is a collection of voters whose preferences are single-peaked with respect to societal axis L , and where $c_1 L c_2 L \dots L c_m$. Within plurality, if $m \geq 2$ then*

1. $\text{score}_{(C, V)}(c_1) = \text{score}_{(\{c_1, c_2\}, V)}(c_1)$,
2. for each i , $2 \leq i \leq m - 1$, $\text{score}_{(C, V)}(c_i) = \text{score}_{(\{c_{i-1}, c_i, c_{i+1}\}, V)}(c_i)$, and
3. $\text{score}_{(C, V)}(c_m) = \text{score}_{(\{c_{m-1}, c_m\}, V)}(c_m)$.

It turns out that this local structure, slightly distorted, still occurs in Dodgson $_k$ -SP societies and in PerceptionFlip $_k$ -SP societies. Thus, our strategy for proving Theorem 17 is to first formally define what we mean by a distorted variant of the above lemma, and then to adapt the CCAC and CCDC algorithms of Faliszewski et al. [23] for single-peaked societies to the distorted setting.

DEFINITION 19. *Let $C = \{c_1, \dots, c_m\}$ be a set of candidates and let L be a linear order over C (the societal*

axis) such that $c_1 L c_2 L \dots L c_m$. For each $c_i \in C$ and each nonnegative integer k , $0 \leq k \leq m - 1$, we define $N(L, C, c_i, k) = \{c_j \mid |i - j| \leq k\}$. We call $N(L, C, c_i, k)$ the k -radius neighborhood of c_i with respect to C and L .

DEFINITION 20. *Let $E = (C, V)$ be a plurality election, let L be a linear order over C (the societal axis), and let k be a positive integer. We say that E is k -local with respect to L if for each $c \in C$ and each $C' \subseteq C$ such that $c \in C'$ it holds that $\text{score}_{(N(L, C', c, k), V)}(c) = \text{score}_{(C', V)}(c)$.*

In particular, the proof of Lemma 18 (given in [23]) shows that single-peaked plurality elections are 1-local with respect to the societal axis. We extend this result to Dodgson $_k$ -SP societies and to PerceptionFlip $_k$ -SP societies.

LEMMA 21. *Let k be a positive integer, let $E = (C, V)$ be a plurality election, and let L be some linear order over C (the societal axis).*

- (a) *If E is Dodgson $_k$ -SP with respect to L then E is $(k+1)$ -local with respect to L .*
- (b) *If E is PerceptionFlip $_k$ -SP with respect to L then E is $(k+1)$ -local with respect to L .*

PROOF. Cases (a) and (b) are similar but not identical and thus we will handle each of them separately.

Case (a) Let $E = (C, V)$ be a Dodgson $_k$ -SP election, where $C = \{c_1, \dots, c_m\}$ and where the societal axis L is such that $c_1 L c_2 L \dots L c_m$. Since for every $C' \subseteq C$, (C', V) is Dodgson $_k$ -SP with respect to L and since E was chosen arbitrarily, it suffices to show that for each candidate $c_i \in C$ it holds that $\text{score}_{(N(L, C, c_i, k+1), V)}(c_i) = \text{score}_{(C, V)}(c_i)$.

Fix a candidate c_i in C and a voter v in V . By definition of Dodgson $_k$ -SP elections, there exists a vote v' that is single-peaked with respect to L , such that v can be obtained from v' by at most k swaps. Let $C' = N(L, C, c_i, k + 1)$. We will show that $\text{score}_{(C', \{v\})}(c_i) = \text{score}_{(C, \{v\})}(c_i)$.

First, if v ranks c_i on top among all candidates in C , then certainly v ranks c_i on top among candidates in C' . Thus, $\text{score}_{(C', \{v\})}(c_i) \geq \text{score}_{(C, \{v\})}(c_i)$. It remains to show that if v does not rank c_i on top among candidates in C , then v also does not rank c_i on top among candidates in C' . We consider two cases:

1. The peak of v' is not in C' . Then it is easy to verify that some candidate from C' precedes c_i in v . Otherwise, to convert v' to v one would need enough swaps for c_i to pass $k + 1$ candidates from C' (either those "to the left" of c_i in L if the peak of v' was "to the left" of c_i , or those "to the right" of c_i if the peak of v' was "to the right" of c_i).
2. The peak of v' is in C' . If v 's top-ranked candidate is in C' then clearly c_i is not ranked first among C' in v . Thus, let us assume that the top ranked candidate in v is not in C' and that it is some candidate c_j . Without loss of generality, let us assume that $j > i + k + 1$ (the case where $j < i - k - 1$ is analogous). Let us also assume that the peak of v' is some candidate c_{i+s} , such that $1 \leq s \leq k + 1$ (the case when $-k - 1 \leq s \leq 0$ is impossible because converting v' to v requires at most k swaps). The minimal number of swaps that convert v' to a vote where c_j is ranked first is at least

$(k + 1) - s + 1 = k - s + 2$ (these swaps involve c_j and candidates $c_{i+s}, c_{i+s+1}, \dots, c_{i+k+1}$. The minimum number of swaps in v' that ensure that c_i is ranked ahead of c_{i+s} is at least s (these swaps involve c_i and candidates $c_{i+1}, c_{i+2}, \dots, c_{i+s}$). Thus, the minimum number of swaps of candidates in v' that ensure that c_j is ranked first and that c_i is ranked ahead of c_{i+s} is $k + 2$, which is more than the allowed k swaps. Thus, this situation is impossible. As a result, some candidate from C' is ranked ahead of c_i in v .

Thus, we have shown that if c_i is not ranked first in v among the candidates from C , then c_i is not ranked first in v among the candidates from C' . Thus, $\text{score}_{(C',\{v\})}(c_i) \leq \text{score}_{(C,\{v\})}(c_i)$, and with the previously shown inequality $\text{score}_{(C',\{v\})}(c_i) \geq \text{score}_{(C,\{v\})}(c_i)$, it must be the case that $\text{score}_{(C',\{v\})}(c_i) = \text{score}_{(C,\{v\})}(c_i)$. Since v was chosen arbitrarily, we have that $\text{score}_{(C',V)}(c_i) = \text{score}_{(C,V)}(c_i)$. This completes the proof of part (a) of the theorem.

Case (b) Let $E = (C, V)$ be an election, where $C = \{c_1, \dots, c_m\}$. Let us assume, without loss of generality, that E is PerceptionFlip $_k$ -SP via societal axis L , where $c_1 L c_2 L \dots L c_m$. Since for every $C' \subseteq C$, (C', V) is PerceptionFlip $_k$ -SP with respect to L and since E was chosen arbitrarily, it suffices to show that for each candidate $c_i \in C$ it holds that $\text{score}_{(N(L,C,c_i,k+1),V)} = \text{score}_{(C,V)}(c_i)$.

Let us fix a candidate c_i in C and a voter v in V . This voter's preference order is single-peaked with respect to some order L' that can be obtained from L by at most k swaps of adjacent candidates. Let us assume that $c_{j'}$ is the candidate directly preceding c_i in L' and $c_{j''}$ is the candidate directly succeeding c_i in L' (in this proof we omit the easy-to-handle cases where c_i is either the maximum or the minimum element of L').

We claim that for any $C' \subseteq C$ that includes $c_{j'}$, c_i , and $c_{j''}$, it holds that $\text{score}_{(C',\{v\})}(c_i) = \text{score}_{(C,\{v\})}(c_i)$. This is so because any voter that ranks c_i on top, ranks c_i on top irrespective of which other candidates are included. So, $\text{score}_{(C',\{v\})}(c_i) \geq \text{score}_{(C,\{v\})}(c_i)$. On the other hand, by Lemma 3.4 of [23], $\text{score}_{(\{c_{j'},c_i,c_{j''}\},\{v\})}(c_i) = \text{score}_{(C,\{v\})}(c_i)$. Thus, any voter that does not rank c_i on top, gives a choice between c_i , $c_{j'}$, and $c_{j''}$ rank one of $c_{j'}$, $c_{j''}$ on top. It is easy to see that $\{c_i, c_{j'}, c_{j''}\} \subseteq N(L, C, c_i, k + 1)$, and so, $\text{score}_{(C',\{v\})}(c_i) \leq \text{score}_{(C,\{v\})}(c_i)$. Thus, $\text{score}_{(C',\{v\})}(c_i) = \text{score}_{(C,\{v\})}(c_i)$ and since we picked v arbitrarily, $\text{score}_{(C',V)}(c_i) \leq \text{score}_{(C,V)}(c_i)$. The proof of case (b) is complete. \square

Now, Theorem 17 is a consequence of the following, more general, result.

THEOREM 22. *For each constant k , CCAC and CCDC for plurality elections are in P for k -local elections.*

However, we do have NP-completeness if in Dodgson $_k$ -SP societies or PerceptionFlip $_k$ -SP societies we allow the parameter k to increase to $m - 2$, where m is the total number of candidates involved in the election. (This many swaps allow us to, in effect, use the same technique as for swoon-SP societies.)

THEOREM 23. *CCAC and CCDC for plurality elections are NP-complete for Dodgson $_{m-2}$ -SP societies and for PerceptionFlip $_{m-2}$ -SP societies, where m is the total number of candidates involved in the election.*

Finally, we note that for single-caved elections, CCAC and CCDC are in P for plurality.

THEOREM 24. *CCAC and CCDC for plurality elections are in P for single-caved societies.*

5. BRIBERY

We now briefly look at bribery of nearly single-peaked electorates, focusing on approval elections. For all three cases—bribery, negative-bribery, and strongnegative-bribery—in which general-case NP-hardness bribery results have been shown to be in P for single-peaked societies [4], we show that the complexity remains in polynomial time even if the number of mavericks is logarithmic in the input size.

THEOREM 25. *Bribery, negative-bribery, and strongnegative-bribery for approval elections over log-maverick-SP societies are each in P, in both the standard and the marked model.*

We mention in passing that although plurality bribery has never been discussed with respect to single-peaked (or nearly single-peaked) electorates, it is not hard to see that the two NP-complete bribery cases for plurality (plurality-weighted- $\$$ bribery and plurality-weighted-negative-bribery) remain NP-complete on single-peaked electorates, in one case immediately from a theorem of, and in one case as a corollary to a proof of, Faliszewski et al. [17].

6. RELATED WORK

Although it has roots going even further back, the study of the computational complexity of control and manipulation actions was started by a series of papers of Bartholdi, Orlin, Tovey, and Trick around 1990 [2, 1, 3] and the complexity of bribery was first studied far more recently [17]. For further references, history, context, and results regarding all of these, see the surveys [18, 21]. For example, it is known that there exist election systems that are resistant to many control attacks [13, 10, 14, 20, 28].

The four papers most related to the present one are the following. Walsh [35] insightfully raised the idea that general complexity results may change in single-peaked societies. His manipulative-action example (STV) actually provides a case where single-peakedness fails to lower manipulation complexity, but in a different context he did find a lowering of complexity for single-peakedness. The papers [19, 4] then broadly explored the effect of single-peakedness on manipulative actions. These three papers are all in the model of (perfect) single-peakedness. Conitzer [6], in the context of preference elicitation, raised and experimentally studied the issue of *nearly* single-peaked societies. Escoffier et al. [15] also discussed nearness to single-peakedness, and the papers [23, 4] both raise as open issues whether shield-evaporation (complexity) results for single-peakedness will withstand near-single-peakedness. The present paper seeks to bring the “nearly single-peaked” lens to the study of manipulative actions.

It is well-known (see Elkind et al. [9] and the references therein) that many election systems are defined, or can be equivalently defined, as selecting whichever candidate's region of being a winner under some notion of “consensus” has a vote set that is “closest” to the input vote set, where “closest” is defined by applying some norm (e.g., sum or max)

to a vector whose i th component is some notion of distance (e.g., number of adjacent-swaps to get between the votes) between the i th votes on each list. We note that, similarly, most of our notions of nearness to single-peaked can be framed as saying that the input vote set is close (in the same sense) to some vote set that is consistent with the input societal linear order. The parallel isn't perfect, since in the former work there are multiple target regions and the minimum over them is crucially selecting the winner; and also, approaches focusing on distance typically require commutativity of the distance function, but notions of diverging from a societal order may, as a matter of human behavior, be asymmetric. (The swoon notion is asymmetric; often one can swoon from v_1 to v_2 , but not vice versa). But we mention that most of our norms/distances have already proven useful in other election contexts, and that if one finds additional norms/distances natural here, then one could study what happens under those.

This paper is focused on the line of work that looks at whether single-peakedness removes NP-hardness results about manipulative actions. *Most of our key results are cases where we show that even nearly-single-peaked elections fall to P. Our results of that sort are polynomial-time upper bounds, and so apply on all inputs.* However, a few of our results are about NP-hardness. For those, it is important to mention that NP-hardness is a worst-case theory, and so such results are just a first step on a path that one hopes may eventually reach some notion of average-case hardness. That is a long-term and difficult goal, but has not yet been proven impossible. Although many people have the impression that recent results such as Friedgut et al. [24] prove that any reasonable election system can often be manipulated, that powerful paper, for example, merely proves that the manipulation probability *cannot go very quickly to zero* asymptotically—it does not prove that the manipulation probability asymptotically does not go to zero. Other work that controls the candidates-to-voters cardinality relation has experimentally suggested stronger claims in certain settings, but is of necessity within the setting of making assumptions about the distribution of votes (see, e.g., Walsh [36]), and typically presents simulations but not theorems and proofs.

Finally, given the high hopes of many people for heuristic algorithms, it is extremely important to understand that it follows from known complexity-theoretic results that (unless shocking complexity-class collapses occur) *no polynomial-time algorithm can come too close to accepting any NP-hard set*. In particular, if any polynomial-time heuristic correctly solves any NP-hard problem on all but a sparse (i.e., at most a polynomial number of strings of each length) set of inputs, then $P = NP$ ([34]; in the terminology of that paper, if any NP-hard—with respect to many-one polynomial-time reductions, of course—set is P-close, then $P = NP$). And so such extremely good heuristic algorithms almost certainly do not exist. But what about less ambitious hopes? Can any polynomial-time algorithm agree with any NP-hard set except for at most $n^{\log^{O(1)} n}$ strings at each length, i.e., the symmetric difference between the set accepted by the algorithm and the NP-hard set has density $n^{\log^{O(1)} n}$ (equivalently, $2^{\log^{O(1)} n}$)? The answer is “no,” unless shocking complexity-theoretic collapses occur. To see this, we simply need to note that if such an algorithm existed, then

the NP-hard set would polynomial-time 1-truth-table reduce (indeed, it would even reduce by a polynomial-time 1-truth-table reduction that was in addition restricted to a single truth-table, namely, the parity truth-table) to a set of density $n^{\log^{O(1)} n}$. However, it is known ([29, 5]) that if any NP-hard set even polynomial-time $\mathcal{O}(1)$ -truth-table reduces (i.e., polynomial-time bounded-truth-table reduces) to a set of density $n^{\log^{O(1)} n}$, then (i) all NP sets can be deterministically solved in time $n^{\log^{O(1)} n}$, and (ii) $EXP = NEXP$ (where $EXP = \bigcup_{\text{polynomials } p} DTIME[2^{p(n)}]$ and $NEXP = \bigcup_{\text{polynomials } p} NTIME[2^{p(n)}]$), i.e., deterministic and nondeterministic exponential time coincide. Both of these consequences are broadly believed not to hold. (Various additional unlikely collapse consequences follow for the case where we are speaking not merely of NP-hard sets but in fact of NP-complete sets [5], such as the few hard problems discussed in this paper.) So polynomial-time heuristic algorithms are extremely unlikely to be able to come within $n^{\log^{O(1)} n}$ errors-per-length of any NP-hard set.³

7. CONCLUSIONS AND OPEN DIRECTIONS

Motivated by the fact that real-world electorates are unlikely to be flawlessly single-peaked, we have studied the complexity of manipulative actions on nearly single-peaked electorates. We observed a wide range of behavior. Often, a modest amount of non-single-peaked behavior is not enough to obliterate an existing polynomial-time claim. We find this the most important theme of this paper—its “take-home message.” So if one feels that previous polynomial-time manipulative-action algorithms for single-peaked electorates are suspect since real-world electorates tend not to be truly single-peaked but rather nearly single-peaked, our results of this sort should reassure one on this point—although they are but a first step, as the paragraph after this one will explain. Yet we also found that sometimes allowing even one deviant voter is enough to raise the complexity from P to NP-hardness, and sometimes allowing any number of deviant voters has no effect at all on the complexity. We also saw cases where frequency of mavericity extracted a price in terms of amount of nondeterminism used. We feel this is a connection that should be further explored, and regarding Corollary 11, we particularly commend to the reader's attention the issue of proving completeness for—not merely

³One may wonder how close to NP-hard sets *can* polynomial-time heuristic algorithms easily come? By using easy “padding” constructions, it is not hard to see that for each positive integer k it holds that every NP-hard set is polynomial-time equivalent to an NP-hard set that differs from the empty set—which itself certainly is in P—on at most $2^{n^{1/k}}$ of the 2^n length- n strings, i.e., they agree on all but an exponentially vanishing portion of the domain; see the construction in Footnote 10 of [12] for how to do this, except with the “2” there being changed to $k + 1$ (see also the comments/discussion in Appendix C of [11]). The best way to interpret this is not to just say that all NP-hard sets are akin to easy sets, but rather to realize that frequency of easiness is not a robust concept with respect to polynomial-time reductions, and neither is average-time complexity computed simply by averaging. Indeed, this type of effect is why average-case complexity theory is defined in ways far more subtle and complex than just taking a straightforward averaging of running times [32].

membership in—the levels of the beta hierarchy. We conjecture that completeness holds.

One might wish to study other notions of closeness to single-peakedness and, in particular, one might want to combine our notions. Indeed, in real human elections, there probably are both mavericks and swooners, and so our models are but a first step. In addition, the types of nearness that appear in different human contexts may differ from each other, and from the types of nearness that appear in computer multiagent systems contexts. Models of human/multiagent-system behavior, and empirical study of actual occurring vote sets, may help identify the most appropriate notions of nearness for a given setting.

Our control work studies just one type of control-attack at a time. We suspect that many of our polynomial-time results could be extended to handle multiple types of attacks simultaneously, as has recently been explored (without single-peakedness constraints) by Faliszewski et al. [16], and we mentioned in passing one result for which we have already shown this.

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APPENDIX A. FORMAL DEFINITIONS

In this section we define our problems formally.

DEFINITION 26 ([7]). *Let R be an election system. In the CCWM problem for R we are given a set of candidates C , a preferred candidate $p \in C$, a collection of nonmanipulative voters S (each vote consists of a preference order and a positive integer, the weight of the vote), and a collection*

T of n manipulators, each specified by its positive integer weight. We ask if it is possible to set the preference orders of the manipulators in such a way that p is a winner of the resulting R election $(C, S \cup T)$.

The following definition is based on those due to Bartholdi et al. [3], except for the case of CCAC, where we follow the more recent approach of Faliszewski et al. [20].

DEFINITION 27 ([3]). *Let R be an election system.*

In the CCAC problem for R we are given two disjoint sets of candidates, C and A , a collection V of votes over $C \cup A$, a candidate $p \in C$, and a nonnegative integer K . We ask if there is a set $A' \subseteq A$ such that (a) $\|A'\| \leq K$, and (b) p is a winner of R election $(C \cup A', V)$.

In the CCDC problem for R we are given an election (C, V) , a candidate $p \in C$, and a nonnegative integer K . We ask if there is a set $C' \subseteq C$ such that (a) $\|C'\| \leq K$, (b) $p \notin C'$, and (c) p is a winner of R election $(C - C', V)$.

In the CCAV problem for R we are given a set of candidates C , two collections of voters, V and W , over C , a candidate $p \in C$, and a nonnegative integer K . We ask if there is a subcollection $W' \subseteq W$ such that (a) $\|W'\| \leq K$, and (b) p is a winner of R election $(C, V \cup W')$.

In the CCDV problem for R we are given an election (C, V) , a candidate $p \in C$, and a nonnegative integer K . We ask if there is a collection V' of voters that can be obtained from V by deleting at most K voters such that p is a winner of R election (C, V') .

The bribery notions below are due to Faliszewski et al. [17], except the notion below of negative and strongnegative bribery for approval voting are due to Brandt et al. [4].

DEFINITION 28 ([17, 4]). *Let R be an election system. In the weighted- $\$$ bribery problem for R we are given an election (C, V) , where each vote consists of the voter’s preferences (as appropriate for the election system, e.g., an approval vector for approval voting and a preference order for plurality) and two integers (this vote’s positive integer weight and this vote’s nonnegative integer price), a candidate $p \in C$, and a nonnegative integer K (the allowed budget). We ask if there is a subcollection of votes, whose total price does not exceed K , such that it is possible to ensure that p is an R -winner of the election by modifying the preferences of these votes.*

The problems (a) weighted-bribery, (b) $\$$ bribery, and (c) bribery for R are variants of weighted- $\$$ bribery for R where, respectively: (a) no prices are specified and each vote is treated as having unit cost, (b) no weights are specified, and each vote is treated as having unit weight, and (c) no prices or weights are specified, and each vote is treated as having unit price and unit weight.

For plurality, “negative” bribery means no bribed voter can have p as the most preferred candidate in his/her preference order.

For approval voting, “negative” bribery means a bribe cannot change someone from disapproving of p to approving of p , and “strongnegative” bribery means every bribed voter must end up disapproving of p .

DEFINITION 29. *For score-based election systems (e.g., plurality, approval, scoring protocols), we write $\text{score}_{(C, V)}(c)$ to denote the score of candidate c in election (C, V) ; naturally we require that $c \in C$. The particular election system that we use will always be clear from context.*