

Some Philosophical Aspects of Reasoning about Knowledge

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The first part of the talk will highlight the way in which assumptions widespread in the literature on reasoning about knowledge generate the traditional philosophical problem of scepticism. The problematic assumptions to be discussed here do *not* include logical omniscience, but are equally serious. My particular concern will be with the so-called Brouwerian axiom: $\sim p \downarrow K\sim Kp$ (if p is false, an agent knows that she does not know p). This axiom follows from the ‘negative introspection’ axiom $\sim Kp \downarrow K\sim Kp$ and the ‘truth’ axiom $Kp \downarrow p$. It corresponds to the condition that the accessibility relation for knowledge be symmetric, and holds on the partition model of knowledge. Let s be a situation in which an agent gains some knowledge p of the external world (Kp), believes that she does so (BKp) and has consistent beliefs (so $\sim B\sim Kp$). Let s^* be a situation in which p is false ($\sim p$) but the input from the external world to the agent is the same as in s , so that the agent has the same beliefs in s^* as in s . Thus $\sim B\sim Kp$ also holds in s^* . Given that knowledge implies belief, it follows that $\sim K\sim Kp$ also holds in s^* , and therefore that the Brouwerian axiom fails in s^* . A defender of the Brouwerian axiom must deny that there can be such pairs of situations s and s^* , but this is tantamount to denying that there can be knowledge of the external world. In fact the outlook for the axiom is even worse, since a similar argument shows it to imply that an agent who has consistent beliefs and believes only when he believes that he knows ($Bp \downarrow BKp$) has no false beliefs: but obviously one cannot guarantee the truth of one’s beliefs about the external world merely by satisfying conditions of internal coherence. Computational problems associated with the axiom will also be briefly mentioned (some connected with Gödel’s second incompleteness theorem and some with agents with limited ability to survey the contents of their own memories). Since the truth axiom is basic to the meaning of ‘knowledge’, and negative introspection implies the Brouwerian axiom given truthfulness, the failure of the Brouwerian axiom implies the failure of negative introspection (‘positive introspection’, $Kp \downarrow KKp$, is problematic too but will not be discussed in this talk).

In the context of epistemic logic, the obvious response to these sceptical problems is to employ models with non-symmetric accessibility relations. Such models have some puzzling properties. In particular, on every finite model with an accessibility relation that is serial (which corresponds to consistency) but not symmetric there is a regular probability distribution such that the prior probability of some proposition can differ from its expected posterior probability (conditionalized on what is known). This in turn leads to Dutch Book problems. A simple example will be discussed. Although this difficulty makes it very tempting to revert to a partition model of knowledge in which expected posterior probabilities always coincide with priors, such a move involves a failure to engage with the fallibility of our beliefs. One challenge faced by reasoning about knowledge is to develop an adequate theory of updating on new knowledge under uncertainty about what one’s new knowledge is.

Some themes of the talk are elaborated in Timothy Williamson, *Knowledge and its Limits* (Oxford: Oxford University Press, 2000).