Dynamic Models of Coalition Formation: Fallback Vs. Build-Up

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Abstract. Players are assumed to rank each other as coalition partners. Two processes of coalition formation are defined and illustrated:

- *Fallback (FB)*: Players seek coalition partners by descending lower and lower in their preference rankings until some majority coalition, all of whose members consider each other mutually acceptable, forms.
- *Build-up (BU)*: Same descent as FB, except only majorities whose members rank each other highest form coalitions.

BU coalitions are *stable* in the sense that no member would prefer to be in another coalition, whereas FB coalitions, whose members need not rank each other highest, may not be stable. BU coalitions are bimodally distributed in a random society, with peaks around simple majority and unanimity; the distributions of majorities in the US Supreme Court and in the US House of Representatives follow this pattern. Other examples of real-life coalition processes are discussed.

1. Introduction

Coalitions are collections of players. Their stability is usually defined in terms of outcomes and the incentives coalition members have to sustain them. In this paper, we show that the process by which the players come together and form coalitions also may critically affect how enduring coalitions will be.

To determine which coalitions are likely to form and be stable, we assume that all players, named 1, 2, ..., n, strictly rank each other as coalition partners, as illustrated in Example A.

Example A. **1**: 2 3 4 5 **2**: 1 3 4 5 **3**: 4 5 2 1 **4**: 3 2 1 5 **5**: 4 3 2 1.

We further assume that each player ranks itself first—that is, it most desires to be included in any majority coalition that forms. In Example A, player 1, after itself, most prefers player 2 as a coalition partner, followed by players 3, 4, and 5 in that order. A complete listing of all players' preferences, as illustrated in Example A, is called a *preference profile*.

It is clear that if there are n players, there are $[(n-1)!]^n$ preference profiles. In our model of a random society used later, all preference profiles are assumed to be equiprobable.

Sometimes we will assume that the players can be placed along a line—in order 1, 2, 3, ..., n, from left to right—so that the preference profile is single-peaked. That is, each player's preference for coalition partners declines monotonically to the left and right of its position in this ordering. A preference profile that satisfies this condition is called

[1] This is an abbreviated version of Brams, Jones, and Kilgour (2003), which contains proofs of all the propositions stated herein and also an illustrative application of the models to two US Supreme Court cases.

ordinally single-peaked (Brams, Jones, and Kilgour, 2002). Such profiles are commonly assumed in spatial models of candidate and party competition.

To express single-peakedness in another way, consider the set of players in a coalition; call the left-most player l and the right-most player r. The set is *connected* if it is of the form $\{l, l+1, \ldots, r\}$: It contains exactly the players from l to r, inclusive. Then a preference profile is single-peaked if and only if, for each $k = 1, 2, \ldots, n$, every player's k most-preferred coalition partners, including itself, form a connected set. Thus in Example A, when $k = 3$, the most-preferred 3-coalitions of players 1 (123), 2 (213), 3(345), 4 (432), and 5 (543) are all connected sets. For all other k between 1 and 5, it is easy to see that all most-preferred k-coalitions are connected, so the preference profile of Example A is ordinally single-peaked.

In fact, such a preference profile may or may not be geometrically realizable in the following sense: If n points can be positioned along a line such that a player's preference decreases as distance from its position increases, then the preference profile is called *cardinally single-peaked* (Brams, Jones, and Kilgour, 2002). To see that this condition is not satisfied in Example A, assume that player i is located at position p_i on the line. Define the distance between two positions, p_i and p_j, to be $d_{ij} = |p_i - p_j|$. From player 3's preference ordering, $d_{54} < d_{53} < d_{32}$, whereas from player 4's ordering, $d_{32} < d_{42} < d_{54}$. This contradiction shows that the preference profile of Example A is ordinally but not cardinally single-peaked.[2]

2. The Fallback and Build-Up Processes

To rule out strategic issues that arise because of differences in player size or capability,[3] we assume that (i) all players are of equal weight (as in a legislature in which each member has one vote) and (ii) winning coalitions are those with at least a simple majority, m, of members. The *fallback (FB)* process of coalition formation unfolds as follows (Brams, Jones, and Kilgour, 2002; Brams and Kilgour, 2001):

1. The most preferred coalition partner of each player is considered. If two players mutually prefer each other, and this is a majority of players, then this is the majority coalition that forms. The process stops, and we call this a level 1 majority coalition because only first-choice partners are considered.
2. If there is no level 1 majority coalition, then the next-most preferred coalition partners of all players are also considered. If there is a majority of players that mutually prefer each other at this level, then this is the majority coalition (or coalitions) that forms. The process stops, and we call this a level 2 majority coalition.
3. The players successively descend to lower and lower levels in their reported rankings until a majority coalition (or coalitions), all of whose members mutually prefer each other, forms *for the first time*. The process stops, with the set of *largest* majority coalition(s)—not contained in any others at this level—designated FB_1.

[2] In other words, the players' ordinal rankings are inconsistent with every possible cardinal representation of their positions.
[3] For example, two ideologically distant players might join together if it would enable them to win, but neither would join a smaller more centrally located player if the resulting coalition were not winning.

Both this process, and the second coalition-formation process (build-up) we will describe shortly, are driven by the players' mutual preferences, not their evaluations of coalitions.[4]

What does FB yield in Example A? At level 1, observe that player 1 prefers player 2, and player 2 prefers player 1, so we designate 12 as a level 1 coalition, as is coalition 34 also.[5] Descending one level, player 3 likes player 5 and player 5 likes player 3, yielding 35 as a coalition at level 2. Descending one more level, majority coalitions 124 and 234 form for the first time: Each player in these coalitions finds the other two players acceptable at level 3. In summary, we have the following coalitions at each level:

Level 1: 12, 34   *Level 2*: 35   *Level 3*: 124, 234.

Notice that coalitions are listed at the level at which they form, except that subcoalitions are never listed. Thus at level 3, pairs 14, 23, and 24 form but do not appear in our listing, because they are proper subsets of coalitions 124 or 234.

Since coalitions 124 and 234 are the first majority coalitions to form, the process stops, rendering $FB_1 = \{124, 234\}$. Observe that players 2 and 4 are common to both coalitions; player 2 prefers coalition 124, and player 4 prefers coalition 234. Obviously, players 1 and 3 prefer the coalition of which each is a member.

Despite the players' preferences being single-peaked, one of the two FB_1 coalitions (124) is *disconnected*: There is a "hole" due to the absence of player 3. The reason that player 3 is excluded from coalition 124 is that whereas players 1 and 2 necessarily rank player 3 higher than player 4 (because of single-peakedness), player 3 ranks players 2 and 1 at the bottom of its preference order. In particular, player 3 does not consider player 1 acceptable at level 3, which excludes player 3 from coalition 124.

While FB is grounded in preferences of players for each other, it could as well be based on their preferences for different features that a policy might include. Thus in Example A, assume players rank five features, $\{a, b, c, d, e\}$, in the same way that they do each other. Then at level l player 1 would find feature *a* acceptable, and at level 2 feature *b*; likewise, player 2 would find both *a* and *b* acceptable at level 2. Consequently, at level 2 (rather than level 1) the coalition 12 would form because of the two players' concurrence on both *a* and *b*. In this example, the level at which coalitions form changes, but not their membership, as players switch from ranking each other to ranking policy features.[6]

The *build-up* (BU) process of coalition formation is the same as FB, with one major difference. As players descend to lower and lower levels, coalitions form if and only if two or more players consider each other mutually desirable *and consider players*

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[4] In Cechlárová and Romero-Medina (2000), each player uses its preference rankings of all other players to evaluate coalitions according to two criteria—the most-preferred, and the least-preferred, members that they contain. Other criteria are postulated in an agent-based simulation model in a neural-network framework, wherein political parties seek to attract a majority of players in a spatial voting game (Iizuka, Yamamoto, Suzuki, and Ohuchi, 2002). Related work on coalition-formation models is discussed in Brams, Jones, and Kilgour (2002).

[5] These preferences are truthful; we will consider later the possibility that the players strategically misreport their preferences.

[6] The number of policy features need not match the number of players. If there are more features than players, coalitions will form later than if there are fewer features than players. For examples, see Brams and Kilgour (2001).

not in the coalition less desirable. In other words, all players in a BU coalition rank each other—and no players outside the coalition—highest. In Example A, this yields the following coalitions at each level:[7]

Level 1: 12, 34 *Level 4*: 12345.

At levels 2 and 3, no new BU coalitions form after coalitions 12 and 34 form at level 1. Only at level 4 does the first majority coalition appear; it is the grand coalition, so BU_1 = {12345}, or just 12345. Note that no member would prefer to be in another 5-coalition—there is none!—proving that this majority coalition is not only *stable* but uniquely so.

Compare this outcome with that produced by FB, which gave FB_1 coalitions 124 and 234 at level 3. These coalitions are *semi-stable*: Even though all their members consider each other acceptable at level 3, some members of each coalition consider some excluded players more desirable as coalition partners. For coalition 124, players 1 and 2 prefer excluded player 3 to included player 4; for coalition 234, player 2 prefers excluded player 1 to included players 3 and 4, and player 3 prefers excluded player 5 to included player 2.[8]

Proposition 1. *BU_1 contains a unique stable coalition. If FB_1 forms at the same level as BU_1, $FB_1 = BU_1$. Otherwise, FB_1 forms at a lower level, in which case all FB_1 coalitions are semi-stable and proper subsets of the BU_1 coalition.*

Example A illustrates Proposition 1. Semi-stable FB_1 coalitions 124 and 234 are contained in stable BU_1 coalition 12345. There are no stable majority coalitions smaller than this grand coalition. Our next example illustrates that BU_1 need not be the grand coalition.

Example B. **1**: 2 3 4 5 **2**: 3 4 1 5 **3**: 4 2 1 5 **4**: 1 2 3 5 **5**: 4 3 2 1.

The FB coalitions at each level are:

Level 2: 23, 24 *Level 3*: 1234.

Whereas no two players consider each other mutually acceptable at level 1, at level 2 two pairs do. At level 3, the first majority coalition forms, so FB_1 = {1234}. But this 4-player coalition is also BU_1, because all its members consider each other, and no others, acceptable. Thus in Example B, the FB and BU processes produce exactly the same majority coalition, which is neither minimal nor grand. To be sure, the grand coalition is also stable, but it seems unlikely to form since players 1 - 4 are united in their opposition to player 5, which they all rank last.

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[7] In Brams, Jones, and Kilgour (2002), a different BU process is proposed in a cardinal-utility context. Coalition members fuse into a single player whose position is the average of its members when preferences are defined by points on the real line.

[8] The exclusion of preferred players from a coalition, and its manipulability (section 3), are two indicators of its instability. While "there is only a relatively small number of results that guarantee the existence of a 'stable' coalition structure" (Greenberg and Weber, 1993, p. 60), even fewer models offer insight into the step-by-step processes of coalition formation that may (or may not) contribute to stability.

If $FB_1 \neq BU_1$, smaller FB_1 coalitions, which are semi-stable, form earlier in the descent, only later to be subsumed by a larger BU_1 coalition that is stable. Thus in Example A, semi-stable FB_1 coalitions 124 and 234 are proper subsets of stable BU_1 coalition 12345.

Proposition 2. *If preferences are single-peaked, at least one FB coalition of two players must form at level 1.*

Proposition 3. *If preferences are single-peaked, then (i) FB_1 coalitions may be disconnected, but (ii) BU_1 is connected.*

Example A, with disconnected FB_1 coalition 124, proves (i).

We now turn to the question of whether players can manipulate either the FB or the BU processes to their advantage. FB, as we will see, is vulnerable to manipulation, but BU is quite robust.

3. The Manipulability of FB and BU

Call a process *manipulable* if one player, by reporting a preference ranking different from its true preference ranking, can induce a majority coalition that it prefers.

Proposition 4. *FB is manipulable.*

Consider the following example:

Example C. **1**: 2 3 4 5 **2**: 3 4 1 5 **3**: 2 4 1 5 **4**: 3 5 2 1 **5**: 4 3 2 1.

The FB coalitions at each level are:

Level 1: 23 *Level 2*: 34, 45 *Level 3*: 123, 234 *Level 4*: 12345.

Now assume player 4 misrepresents its preferences as follows:

4: 3 2 5 1.

Then FB gives the following:

Level 1: 23 *Level 2*: 234 *Level 3*: 123 *Level 4*: 12345.

When player 4 is truthful, FB_1 = {123, 234}, whereas when player 4 misrepresents its preferences, FB_1 = {234}. Because player 4 prefers coalition 234 to coalition 123, misrepresentation, which precludes the possibility of coalition 123, is rational, rendering FB manipulable.[9]

[9] Thus, truthful reporting is not a Nash equilibrium under FB, given the strategies of players are to be truthful or not in a noncooperative game (player 4 would have an incentive not to be truthful in Example C). As we will show next, however, a player cannot assuredly do better by misrepresenting its preferences under BU. Thereby *when* the process of coalition formation terminates affects the stability of outcomes

Proposition 5. *BU is not manipulable.*

To illustrate, misrepresentation will not be rational for player 4 if the comparison is between the (apparent) BU_1 coalition that forms with misrepresentation and one that forms without misrepresentation. With misrepresentation, $BU_1 = 234$; without misrepresentation, $BU_1 = 12345$. Because the larger coalition, 12345, includes both a preferred player (5) and a non-preferred player (1) compared to player 2 in the smaller coalition, 234, one cannot say that player 4 prefers 234 to 12345 or vice versa. Thus, by reporting a preference ranking different from its true preference ranking, player 4 cannot induce a majority coalition that it *assuredly* prefers, illustrating the nonmanipulability of BU.[10]

4. Properties of Stable Coalitions

After the appearance of BU_1, larger and larger BU majority coalitions may—or may not—appear at subsequent levels of descent. Each larger BU majority coalition contains all smaller BU majority coalitions, as illustrated next with a cardinally single-peaked example.

Example D. **1**: 2 3 4 5 6 7 **2**: 1 3 4 5 6 7 **3**: 2 1 4 5 6 7 **4**: 3 2 1 5 6 7
5: 6 4 3 2 1 7 **6**: 5 4 3 2 1 7 **7**: 6 5 4 3 2 1.

Geometrically, we can represent the preferences of these players by placing them at points along the real line:

1 2 3 ___ 4 ___ 5 6 ___ 7.

Thus, for example, the members of pairs 12 and 56 are each other's most-preferred coalition partners, for they are closer to each other in distance than to any other players. Because player 3 prefers players 2 and 1 to player 4, player 3 is farther from player 4 than from player 1. Likewise, players 5 and 6 are farther from player 7 than from player 1, because they rank player 7 last.

We list below all the FB coalitions, not contained in any others at each level, distinguishing those that are also BU coalitions:

Level 1: 12 (BU), 56 (BU) *Level 2*: 13, 23 *Level 3*: 1234 (BU)
Level 4: 2345 *Level 5*: 123456 (BU) *Level 6*: 1234567 (BU).

Observe that the first FB majority coalition to appear, 1234 at level 3, is also a BU majority coalition, so $FB_1 = BU_1 = 1234$. As the descent continues, there is no BU

generated under it, underscoring our contention—reflected in the subtitle of our longer paper (see note 1)—that "the process matters."

[10] To be sure, if there were more information about preferences—in particular, cardinal valuations of different coalitions by each player—it would be possible to say whether player 4 prefers 234 to 12345 or vice versa. In the absence of such information, however, we assume that player 4 does *not* have an incentive to depart from reporting its true preference that yields 12345.

coalition at level 4, but at level 5 a 6-member BU coalition forms. Finally, the grand coalition, which is always a BU coalition, appears at level 6.

Given a cardinally single-peaked preference profile, define the *spread* of a coalition to be the distance between its extreme players. Thus, the spread of coalition 1234 is the distance between player 1 on the left and player 4 on the right, or d_{14}. That this distance is less than d_{45} ensures that coalition 1234 forms before player 5 is brought into the fold. But because player 5 ranks player 6 above all other players, player 5 does not find player 1 acceptable at level 4—only players 2, 3, 4, and 6 are acceptable at this level.

Hence, coalition 12345 is not a BU coalition. On the other hand, because the spread of coalition 123456 is less than the distance between player 6 and player 7, coalition 123456 is a BU coalition at level 5, as is the grand coalition, 1234567, at level 6.

If players' preferences are cardinally single-peaked, it is easy to discern the stable coalitions that form from the players' positions and distances between them.

Proposition 6. *If preferences are cardinally single-peaked, then a subset of players is a BU coalition if and only if it is connected and its spread is less than the distances from each extreme member (other than 1 and n) to the nearest player not in the subset.*

Put more informally, a coalition that is disconnected cannot be a BU coalition, because members would rank the left-out member higher than some members of the coalition. Now assume a coalition is connected but that the distance of an extreme member to an adjacent non-member—either on the left or on the right—is less than the spread. Then the adjacent non-member will be ranked higher by the extreme member than some player in the coalition, so the coalition cannot be a BU coalition.

Proposition 6 provides a characterization of BU coalitions if the players have cardinally single-peaked preferences, thereby enabling one to "read" the BU coalitions from the geometric representation. In general, members of a BU coalition must be sufficiently isolated from players outside it to rank only each other tops.

Whether players' preferences are cardinally single-peaked or not, it is always possible to ensure the existence—or nonexistence—of BU majority coalitions at any level from $m - 1$ (simple majority coalition) to $n - 1$ (grand coalition).

Proposition 7. *BU majority coalitions may appear—or not appear—at any level, up to the appearance of the grand coalition.*

Example D illustrates that a less-than-majority BU coalition (56) and a majority BU coalition (1234) can co-exist. However, two different BU majority coalitions, which of necessity overlap, cannot co-exist, as is possible under FB (see Example A for an illustration).

Proposition 8. *If two BU coalitions intersect, then one contains the other.*

A consequence of Proposition 8 is that any majority BU coalition of a specific size is unique. In particular, BU_1 contains only a single coalition, as already noted in

Proposition 1. In Example D, the BU coalitions that form at levels 1 and 3 are contained in the level 5 BU coalition, which in turn is contained in the level 6 BU coalition. But BU coalition 56, which forms at level 1, is disjoint from BU majority coalition 1234 that forms at level 3. In general, if BU coalitions co-exist, then at most one is of majority size.

In section 3, we showed that members of BU_1 cannot, in general, induce a preferred majority coalition, although they might be able to speed up the formation of an apparent (smaller) BU_1 coalition. But what if a non-member of BU_1 desires to be part of a BU coalition? We next show that such a player can conceivably benefit from a *bandwagon strategy*, which enables it, by misrepresenting its preferences, to be part of a larger BU majority coalition sooner than it would be if it were truthful.

To illustrate, suppose that player 5 in Example D reports its preference ranking to be

5: 4 3 2 1 6 7.

At level 5, BU majority coalition 12345 will form, which includes player 5. By comparison, if player 5 were truthful, the next BU majority coalition to form—after BU_1 = 1234—would be 123456. Because player 6 is player 5's most-preferred coalition partner, player 5 does not necessarily benefit from a bandwagon strategy, even though this strategy puts it into a smaller BU majority coalition at level 4 rather than level 5.

If there is a benefit, it would come by misrepresenting one's preferences in order to join the winning coalition early (i.e., "jumping on the bandwagon"). Indeed, there is evidence from US national party conventions of delegates' shifting to the expected winner—allegedly to demonstrate party unity—as soon as the handwriting of victory is on the wall. Such proclamations of support may well be motivated by cold-blooded calculations of the direct benefit (e.g., a government appointment) that sometimes accrues to former opponents (Brams, 1978).

5. The Incidence of Stable Coalitions

Because BU coalitions may or may not exist at every level from simple majority (of size m) to grand, it is useful to ask when they are most likely to form. Let $P(n, k)$, be the probability that a k-coalition ($k \geq m$) is stable if all strict preference rankings of n players are equally likely, which we call a *random society*. The following proposition describes the behavior of this probability as the size of a majority coalition increases from m to n.

Proposition 9. *The probability of a BU coalition, starting at $k = m$, decreases to a minimum at some intermediate value of k before increasing to 1 at $k = n$. More precisely, for each $n \geq 3$, there exists an integer $k_0(n) = k_0$, satisfying $m \leq k_0 < n$, such that $P(n, k + 1) < P(n, k)$ if $m \leq k < k_0$, $P(n, k_0 + 1) \geq P(n, k_0)$, and $P(n, k + 1) > P(n, k)$ if $k > k_0$. Moreover, $k_0(n) > m$ whenever $n \geq 5$.*

For small values of n and k, we have calculated not only $P(n, k)$ but also $Q(n, k)$, the probability that a k-coalition ($k \geq m$) is stable when all preference rankings of the n players are ordinally single-peaked and equally likely to occur. In addition, we have made analogous calculations of the probabilities, $P_1(n, k)$ and $Q_1(n, k)$, that stable majority coalitions form *for the first time*—that is, form at size k but not earlier. All these probabilities are given in Table 1 for values of n between 3 and 9, and all values of k between m and n.

Table 1 about here

Consider the P values in the Table 1A. For fixed n, these probabilities are virtually identical when $n = 7$ and $n = 9$. They first decrease going from $k = m$ to some intermediate value of k, and then increase to almost 1 in the case of $P_1(n, k)$, and to 1 in the case of $P(n, k)$.

What Proposition 9 does not indicate, though the numerical values of both $P(n, k)$ and $P_1(n, k)$ do, is that even when $k = m$, these probabilities are very small compared with their values when $k = n$. In other words, almost all BU_1 coalitions in a random society form—in fact, form for the first time—only when the grand coalition appears.

It is evident that the probability that *any* BU majority coalition (except the grand coalition) forms in a random society becomes vanishingly small as n increases. This reflects the fact that there at most one BU coalition at each majority size, and that stability is a certainty only for the grand coalition.

While the probability values in Table 1 may not be empirically accurate, the *distributions* may be qualitatively correct in many situations. As we will see later, majority coalitions in real-life voting bodies often do cluster around simple majority and grand—that is, their distribution is V-shaped between $k = m$ and $k = n$, as the BU model predicts.

To be sure, the bimodal distribution of the probability values for general preferences concentrates almost all the support on the grand coalition. This support is dampened somewhat if preferences are restricted to profiles that are ordinally single-peaked (see the Q values in Table 1B). When $n = 5$, for example, $Q_1(5, 3) = 0.333$ and $Q_1(5, 4) = 0.104$, compared with $P_1(5, 3) = 0.046$ and $P_1(5, 4) = 0.016$. Thus, in the former case there is a 44% chance that BU_1 will not be the grand coalition, whereas in the latter case there is only a 6% chance.

Of course, coalition formation does not generally occur in a random society. Subsets of players, such as political parties in a national legislature, will have members with similar preferences. In such situations, we would expect less-than-grand coalitions to form more frequently and be stable.

We conjecture that the distribution of FB_1 semi-stable majority coalitions in a random society, for which we have not yet made detailed calculations, is also *V*-shaped, whether preferences are general or ordinally single-peaked. But instead of the *V*'s being so heavily weighted on the side of the grand coalition—that the *V* looks more like a *J*—our preliminary calculations indicate that the FB_1 distribution will be considerably flattened, so there will be more weight in the middle as well as around a simple majority.

We conclude this section with empirical data on the formation of majority coalitions in two venerable American institutions. In the 9-person US Supreme Court, majority coalitions fit the bimodal probability distribution we found under BU, with majorities tending to be either minimal winning or unanimous. Between 1962 and 1997, we have the following distribution (Edelman and Sherry, 2000), with the minimum occurring at majority size 7:[11]

Majority size:	5	6	7	8	9
Percent of cases:	24	21	13	14	27

There are other voting bodies in which our models seem applicable, including the U.S. House of Representatives. Indeed, as in the Supreme Court, there is a bimodal distribution of majority sizes, based on the 12,688 roll call votes between 1955 through 1990:[12]

Percent majority:	50–60	60-70	70-80	80-90	90-100
Percent of roll calls:	26	19	14	11	30

Although the minimum occurs in the 80-90 percent range, not the middle 70-80 percent range, the two modes are the near-majority and near-unanimity ranges, consistent with the BU model.

6. Conclusions

BU seems most applicable to studying coalition formation in multimember courts and legislatures, in which small subsets of members coalesce and build up to a majority, all of whose members rank each other highest and are therefore stable. FB probably better describes the formation of a governing coalition in parliamentary democracies, wherein disconnected coalitions sometimes form. Because parties in such coalitions rank some parties outside the coalition higher than parties in it, these coalitions are at best semi-stable.[13]

Insofar as voters' preferences are single-peaked, the coalition governments that form are usually connected. Indeed, they are often described by such terms as "left-center" or "center-right." On occasion, however, the left and right do get together and form national-unity governments—sometimes in response to a crisis, like the threat of war—in which many members may be far from each other's favorite coalition partners.

Such semi-stable coalitions, which may be disconnected, tend not to last. According to Riker's (1962) size principle, some of their members grow disaffected and leave if there are insufficient resources to reward them in an oversized coalition.

Through manipulation, players can disrupt semi-stable coalitions by announcing false preferences. Not all these changes, however, may be purely opportunistic. For example, Jim Jeffords, a US Senator from Vermont, switched from the Republican party

[11] These data are drawn from Edelman and Sherry (2000), who also note the bimodal character of the Supreme Court majority decisions. They explain it in terms of a Markov process of coalition formation, using the Supreme Court voting data to calculate the probability of different absorbing states. By contrast, we predict the V-shaped distribution of different-size coalitions on theoretical grounds, independent of any data.

[12] We are grateful to Jeffrey E. Cohen for calling our attention to these data, which were compiled by David W. Rohde for the Inter-University Consortium for Political and Social Research in January 1995.

[13] Because the significant players in parliamentary democracies are different-size parties, strategic considerations come into play that the FB and BU do not take account of (see note 3). Data on coalition governments in Western Europe can be found in Müller and Strom (2000).

to become an independent in 2001, turning the Democratic party into the majority party in the Senate. He seems to have been motivated by a genuine belief that he could better serve Vermont and his country by changing his party affiliation. By contrast, we suggested that the preference changes that create bandwagons may not be so sincere.

To conclude, coalition-formation processes affect the size and stability of the coalitions they generate. If stability can be measured by durability, then our models may provide insight into why parliamentary coalitions in a country like Italy are less durable than those in the Scandinavian countries, where government coalitions sometimes do not include even a simple majority of members.

The models might also enhance our understanding of the stability of coalitions in other arenas, including international relations. Some international alliances like NATO have been long-lasting, others ephemeral. Is the process that led to the former more BU-like, the latter more FB-like? Our models, we believe, provide tools for investigating such questions.

References

Brams, Steven J. (1978). *The Presidential Election Game.* New Haven, CT: Yale University Press.

Brams, Steven J., Michael A. Jones, and D. Marc Kilgour (2003). "Forming Stable Coalitions: The Process Matters." Preprint, Department of Politics, New York University.

Brams, Steven J., Michael A. Jones, and D. Marc Kilgour (2002). "Single-Peakedness and Disconnected Coalitions." *Journal of Theoretical Politics* 14, no. 3 (July): 359-383.

Brams, Steven J., and D. Marc Kilgour (2001). "Fallback Bargaining." *Group Decision and Negotiation* 10, no. 4 (July): 287-316.

Cechlárová, Katarina, and Antonio Romero-Medina (2000). "Stability in Coalition Formation Games." *International Journal of Game Theory* 29, no. 4 (December): 487-494.

Edelman, Paul H., and Suzanna Sherry (2000). "All or Nothing: Explaining the Size of Supreme Court Majorities." *North Carolina Law Review* 78, no. 5 (June): 1225-52.

Greenberg, Joseph, and Shlomo Weber (1993). "Stable Coalition Structures with a Unidimensional Set of Alternatives." *Journal of Economic Theory* 60, no. 1: 62-82.

Iizuka, Hiroyuki, Masahito Yamamoto, Keji Suzuki, and Azuma Ohuchi (2002). "Bottom-up Consensus Formation in Voting Games." *Nonlinear Dynamics, Psychology, and Life Sciences* 6, no. 2 (April): 185-195.

Müller, Wolfgang C., and Kaare Strom (eds.) (2001). *Coalition Government in Western Europe.* Oxford, UK: Oxford University Press.

Riker, William H. (1962). *The Theory of Political Coalitions.* New Haven, CT: Yale University Press.

Table 1

Probabilities of Stable (BU) Coalitions (P, Q) and First-Forming Stable (BU_1) Coalitions (P_1, Q_1)

A. All preference profiles equiprobable

	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9
$P(3,k)$	0.75	1						
$P_1(3,k)$	0.75	0.25						
$P(4,k)$		0.1481	1					
$P_1(4,k)$		0.1481	0.8519					
$P(5,k)$		0.0463	0.0195	1				
$P_1(5,k)$		0.0463	0.0166	0.9371				
$P(7,k)$			2.19×10^{-4}	2.77×10^{-5}	1.50×10^{-4}	1		
$P_1(7,k)$			2.19×10^{-4}	2.76×10^{-5}	1.50×10^{-4}	0.9996		
$P(9,k)$				7.50×10^{-8}	2.72×10^{-9}	2.67×10^{-9}	5.36×10^{-7}	1
$P_1(9,k)$				7.50×10^{-8}	2.72×10^{-9}	2.67×10^{-9}	5.36×10^{-7}	0.999999

B. All ordinally single-peaked preference profiles equiprobable

	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9
$Q(3,k)$	1	1						
$Q_1(3,k)$	1	0						
$Q(4,k)$		0.4444	1					
$Q_1(4,k)$		0.4444	0.5556					
$Q(5,k)$		0.3333	0.1875	1				
$Q_1(5,k)$		0.3333	0.1042	0.5625				
$Q(7,k)$			0.0640	0.0640	0.0308	1		
$Q_1(7,k)$			0.0640	0.0591	0.0272	0.8497		
$Q(9,k)$				8.55×10^{-3}	2.75×10^{-3}	2.12×10^{-3}	4.81×10^{-3}	1
$Q_1(9,k)$				8.55×10^{-3}	2.54×10^{-3}	2.01×10^{-3}	4.65×10^{-3}	0.9822