Incorporating Unawareness into Contract Theory

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Abstract

Asymmetric awareness of the contracting parties regarding the uncertainty surrounding them is proposed as a reason for incompleteness in contractual forms. An insurance problem is studied between a risk neutral insurer, who has superior awareness regarding the nature of the uncertainty, and a risk averse insuree, who cannot foresee all the relevant contingencies. The insurer can mention in a contract some contingencies that the insuree was originally unaware of. It is shown that there are equilibria where the insurer strategically offers incomplete contracts. Next, equilibrium contracts are fully characterized for the case where the insuree is ambiguity averse and holds multiple beliefs when her awareness is extended. Competition among insurers who are symmetrically aware of the uncertainty promotes awareness of the insuree.

1 Introduction

Contracting agents may not symmetrically foresee all the contingencies that they will encounter with in the future. For example, insurance companies who have been in the industry for a long time may take into account more realizations of nature than a first time insurance buyer. In such a setting, will the insurer mention in the contract those contingencies that the insuree does not foresee originally, or will he remain silent on them? If the insuree reads a clause about a contingency that did not cross her mind originally, how will she evaluate this information?

An agent is unaware of a state of the world if she does not know it, she does not know that she does not know it, and so on ad infinitum. An agent cannot take into account the contingencies that she is unaware of. We call the set of contingencies that an agent takes into account in her decision making process as her awareness set. We model an insurance setting between an insurer (he) and an insuree (she). The insurer has a superior awareness on the set of relevant contingencies. The insurer offers a take it or leave it contract to the insuree. A contract specifies some set of contingencies at which the damage is going to be covered, the amount of coverage, and the premium. If the contract does not mention some relevant contingencies, then the insurer does not make a transfer when an unmentioned contingency materializes. This kind of contracts are incomplete. While reading a contract, the insuree may become aware of new contingencies and as a result start taking into account those contingencies as well. The question is how she is going to assign probabilities to them.

In standard game theory, the awareness of agents does not evolve throughout the interaction. Therefore, the standard solution concepts in game theory do not provide an equilibrium notion for our setting. The closest equilibrium concept to ours in the literature is introduced by Ozbay (2006). We define equilibrium as a triplet which contains the strategies of the insurer and the insuree, and the probability distributions on the insuree's awareness set after each offer.

A priori, there is no imposition on how the insuree generates a belief when her awareness is extended. Belief formation of the insuree is a part of the equilibrium concept. We progressively require more restrictions on belief formation. We start with compatible belief, then we will consider consistent beliefs, and finally we will focus on the pessimistic beliefs. The definitions of these concepts are given and discussed extensively in the paper.[1] Roughly, we call a belief *compatible with a contract* if with respect to this belief the insuree

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[1] Proofs of all the statements are available in Appendix at http://www.columbia.edu/∼ef2011

thinks that the insurer is better off by making this offer rather than staying out of business. We require equilibrium beliefs to be compatible with the corresponding contracts whenever it is possible. Under this solution concept we show that incomplete contracts are always part of some equilibria while complete contracts may not be. Next, we refine this possibly large equilibrium set with a consistency requirement. A belief held after a contract is *consistent* if the contract is the best one for the insurer according to the insuree with respect to this belief.

Insurance companies provide coverage against high cost contingencies while not mentioning some less costly ones at all. For example, Dell Inc. announces that its warranty will cover against costly accidental damages such as liquid spill on computers or power surges but does not say anything about some cheaper damages such as logical errors. Moreover, Dell Inc. advertises this warranty as "*Expect the worst. Get the plan that's best for you.*" It seems that Dell Inc. aims to appeal to the pessimism of the computer buyers who start taking into account various other contingencies after reading the contract.

With this example in mind, next we model the insuree as a pessimist (ambiguity averse) agent. She cannot pick a probability distribution among the compatible ones corresponding to a contract. Instead, she evaluates a situation by the worst scenario in her multiple belief set generated by the compatible probability distributions corresponding to the offered contract. While dealing with an ambiguity averse insuree, the insurer has two concerns: a) including costly contingencies in the contract makes the insuree evaluate the situation as very harmful and willing to pay high premium, and b) promising insurance at high cost contingencies is costly for the insurer. The trade-off between the two concerns determines the form of equilibrium contract under ambiguity aversion. We show that the insurer offers a contract either only on the contingencies that the insuree initially foresees, or he announces just one contingency in addition to the ones the insuree is already aware of.

By hiding some contingencies, the insurer can manipulate the insuree's evaluation of the situation. He makes the insuree consider only a part of the uncertainty as if it is everything and then he benefits from the premium without making any transfer at those hidden contingencies. Policy makers care about this manipulation in insurance contracts. The doctrine of concealment is applied to encourage utmost good faith in writing insurance contracts (see for example Bertram Harnett, 1950). We show that competition between insurers can be an instrument to extend the insuree's awareness set. When the size of competition is large enough, we show that in the symmetric equilibrium where the insuree buys the contract, each insurer offers a complete, zero profit contract: competition promotes awareness.

In the contracting model we study in this paper, both complete and incomplete contracts are feasible. Incomplete contracts arise as a result of strategic interaction. Therefore, unawareness can lead to endogenous incompleteness in contractual forms.

Based on economic theory of firm by Williamson (1971, 1979), incomplete contracts are mostly addressed in order to analyze institutions.[2] In this literature, the inability of contracting parties to foresee some aspects of the state of the world is frequently understood as a reason for incompleteness of contracts. However, this reasoning lead to well known discussions in the studies of Maskin and Tirole (1999), Tirole (1999) and Maskin (2002). They argued that in the models motivated by unforeseen contingencies, the parties are rational and able to understand the payoff related aspects of the state of the world although they are unable to discuss the physical requirements leading to those payoffs. Tirole (1999) states that the way *they currently stand*, unforeseen contingencies are not good motivation for models of incomplete contracts, and he further notes that:

> "...There is a serious issue as to how parties form probability distributions over payoffs when they cannot even conceptualize the contingencies..., and as to how they end up having common beliefs ex ante. ...[W]e should have some doubts about the validity of the common assumption that the parties to a contract have symmetric information when they sign the contract."

In line with the observation quoted above, in our model the agents cannot forecast the relevant contingencies symmetrically. They are rational agents within their awareness set but they are taking into account only the aspects of the uncertainty that they are able to conceptualize.

Unawareness is first studied in economic theory by Modica and Rustichini (1994). In the literature there are some recent developments in modelling interactive awareness. Interactive awareness models by Heifetz, Meier, and Schipper (2006) and Li (2006) can be thought of as the basis of the awareness concept we use in this paper. In those models, what each agent can take into account is a projection of the entire situation to the aspects she is aware of. This set theoretic

[2] For a summary of this literature, we refer the reader to Bolton and Dewatripont (2005) and Salanié (2005).

modelling of awareness is incorporated into game theory by Halpern and Rego (2006) and Ozbay (2006). As an application, our model is closer to the later reference since we also have communication between agents regarding the nature of the uncertainty through contracts.

Standard economic theory has developed in a paradigm which excludes unawareness. Recent studies addressed how accounting for awareness changes the standard economic theory (see Modica, Rustichini, and Tallon, 1998, and Kawamura, 2005, for applications in general equilibrium models). Although the papers written in both awareness and incomplete contracts literatures always refer to each other, there are not many studies that explicitly conflate two strands of the theoretical literature. Our study can be thought as one of the first attempts in contract theory which formally allows unawareness.

The rest of the paper is organized as follows: In Section 2, we introduce the one insurer-one insuree model and necessary notation. In Section 3, we give an equilibrium concept and study the form of equilibrium contracts that can arise in this setting. Then in Section 4, we model the insuree as a pessimist agent who is unable to hold single belief after each contract but instead presents an ambiguity averse behavior on a multiple belief set. We see that the equilibrium contracts in this case are always incomplete if the insuree is unaware of at least two contingencies. By introducing competition between insurers in Section 5, we show that the unawareness of the insuree can totally disappear under competition. In Section 6, we discuss some key points in the construction of our model. We conclude in Section 7.

2 Model

There is a good owned by an agent. $v > 0$ is the value of the good for the agent. The good is subject to some uncertain future damages. The owner (insuree) wants to insure the good against realization of damages. The cost of a potential damage is an element of S which is a finite subset of positive real numbers.[3] Elements of S are called contingencies and they are distributed according to μ. It is assumed that all the elements of S are possible, i.e. $\forall s \in S$, $\mu(s) \neq 0$.

The insuree (she) is indexed by 0 and we assume that there is only one insurer (he) indexed by 1.[4] If a contingency is in an agent's state of mind while s/he is evaluating a situation, then we say that s/he is *aware* of that contingency. Otherwise, if the agent is unaware of a contingency, then s/he cannot take that contingency into account in the decision making process. The awareness structures of the insurer and the insuree are as follows:

- The insurer is aware of S and believes the distribution μ.
- The insuree is only aware of S', which is a proper subset of S such that $\mu(S') > 0$. She believes the conditional distribution $\mu(.|S')$.
- The insuree is not aware of remaining realizations of damages in $S \backslash S'$ and she is not aware of the insurer's superior awareness. Therefore, initially the insuree believes that $(S', \mu(.|S'))$ describes the whole uncertainty that she and the insurer consider.
- The insurer knows that the insuree is considering only $(S', \mu(.|S'))$ and moreover, the insurer knows that the insuree is unaware that the insurer has superior awareness.

Given this awareness structure, each party interprets the true problem as a projection of it onto the aspects of the uncertainty that s/he is aware of.

The set of contingencies an agent is aware of can be thought of as the contingencies that that agent experienced during his/her past interactions with nature which is not modeled here. The initial beliefs are developed by these experiences. Since contingencies are distributed by μ, each agent believes the true distribution conditional on the set of contingencies that s/he is aware of initially in this model. This explains why the initial belief of each agent is determined by μ.

The insurer offers a contract in order to insure the good against future damages. A typical contract is a specification of three objects:

(i) The contingencies on which a money transfer will be made from insurer to insuree;

(ii) The amount of transfer as a function of contingencies in (i);

(iii) The premium which is an in advance payment from insuree to insurer for the agreement.

Definition 1 *A contract is a triplet* $C = (t, A, k)$ *where* $A \subseteq S$, $t : A \rightarrow \mathbb{R}_+$ *is the transfer rule, and* $k \in \mathbb{R}_+$ *is the premium. The set of all contracts is denoted by* $\mathbb{C}$.

[3] The tools we develop here can be easily modified for infinite S.

[4] Until Section 5 we assume that there is only one insurer. Then we will introduce competition in the model.

We consider only the transfer rules that cannot promise a transfer larger than the cost of damage at any realization of the uncertainty. Otherwise, the insurer can make infinite amount of profit.

Note that Definition 1 does not restrict the set of contingencies that a contract can be written on. A can be any subset of S. If the contract is silent at some contingencies, it means that there will not be any transfer to the insuree when those contingencies are realized.[5]

Definition 2 *A contract $C = (t, A, k)$ is incomplete if $A \cup S' \neq S$.*

If a contract mentions more contingencies than the insuree takes into account initially then the insuree becomes aware of those contingencies and her new understanding of the uncertainty enlarges to the aspects in $A \cup S'$. This means that there is no language barrier and the insuree is capable of understanding the content of the offer. Since there are contingencies that the insuree is not aware of, as long as a contract does not mention them, the insuree will remain unaware of them and continue to omit these contingencies in her decision making process.

When a contract $C = (t, A, k)$ is offered, the insuree holds a belief P_C which is a probability distribution on $A \cup S'$. The way beliefs are generated is a part of our solution concept and starting from Section 3, we will analyze the relationship between the formation of belief and the form of signed contracts. Here we will introduce the necessary notation only for an arbitrary belief P_C.

After a contract C is offered, the insuree can either reject or take the offer. If she rejects the offer, then the negotiation stops at that point and she is not covered for any damage. The decision of the insuree on a contract is determined by a function $D : \mathbb{C} \to \{buy, reject\}$.

We assume that the insuree is a risk averse agent with an increasing and concave utility function u. Therefore, the expected utility of the insuree from contract $C = (t, A, k)$ with respect to distribution P_C can be written as

$$EU_0(C, D(C)|P_C) := \begin{cases} \sum\limits_{s\in A} u(v - s + t(s) - k)P_C(s) & \text{if } D(C) = buy \\ \quad + \sum\limits_{s\in S'\setminus A} u(v - s - k)P_C(s) & \\ & \\ \sum\limits_{s\in A\cup S'} u(v - s)P_C(s) & \text{if } D(C) = rej. \end{cases}$$

[5]We will discuss in Section 6 some other types of contracts that we did not consider here because they would not change the results.

The expected utility of the risk neutral insurer from contract $C = (t, A, k)$ is

$$EU_1(C, D(C)) := \begin{cases} k - \sum\limits_{s\in A} t(s)\mu(s) & \text{if } D(C) = buy \\ 0 & \text{if } D(C) = reject \end{cases}$$

Observe that the expected utility of the insurer calculated by the insurer himself and the one calculated by the insuree under her belief P_C may not coincide in general. The insurer's expected utility from contract C according to the insuree with respect to her belief P_C is denoted by

$$EU_1^0(C, D(C)|P_C) := \begin{cases} k - \sum\limits_{s\in A} t(s)P_C(s) & \text{if } D(C) = buy \\ 0 & \text{if } D(C) = reject \end{cases}$$

3 Incompleteness in the Contractual Form

The crucial and non-standard point in our model is the following: although before anything is offered, the insuree is unaware of some relevant aspects of the uncertainty, once they are announced to her via a contract, she starts taking them into account. Her awareness evolves throughout the interaction. The contracts that extend the awareness set of the insuree do not inform her regarding the probability of those newly announced contingencies. However, the content of the contract might still be informative about the probability of contingencies it specifies. When contract $C = (t, A, k)$ is offered, the insuree needs to generate a belief which is a probability distribution on her extended awareness set, $A \cup S'$.

Definition 3 *A probability distribution $P_C \in \Delta(A \cup S')$ is compatible with contract $C = (t, A, k)$ if it satisfies:*

(i) $EU_1^0(C, buy|P_C) \geq 0$, i.e. $k \geq \sum_{s\in A} t(s)P_C(s)$;

(ii) For any $s \in A \cup S'$, $P_C(s) \neq 0$ and $P_C(.|S') = \mu(.|S')$

The set of all probability distributions that are compatible with C is denoted by Π_C.

Compatible beliefs are candidates to be held by the insuree after an offer. The solution concept we introduce in this section requires that belief formation is a part of an equilibrium. The insurer believes that the insuree will behave according to the beliefs that an equilibrium suggests under some rationality requirements and he responses to this belief. Equilibrium behavior of the insuree confirms this belief of the insurer as well.

Definition 4 *An equilibrium of this contractual model is a triplet* $(C^*,\ D^* : \mathbb{C} \rightarrow \{buy, reject\}, (P^*_C)_{C\in\mathbb{C}})$ *such that*

(i) $C^* \in \underset{C\in\mathbb{C}}{\arg\max} EU_1(C, D^*(C))$;

For any $C \in \mathbb{C}$,

(ii) $$D^*(C) = \begin{cases} buy & \text{if } EU_0(C, buy|P^*_C) \\ & \quad \geq EU_0(C, reject|P^*_C) \text{ and } \Pi_C \neq \emptyset \\ reject & \text{otherwise} \end{cases}$$

(iii) *for any* $s \in A \cup S'$, $P^*_C(s) \neq 0$, $P^*_C(.|S') = \mu(.|S')$, *and* $P^*_C \in \Pi_C$ *whenever* $\Pi_C \neq \emptyset$.

Observe that the set of compatible probability distributions with a contract is empty only if the premium charged is smaller than all the transfers promised by the contract. We may interpret such contracts as *too good to be true* offers. With respect to any probability distribution on the insuree's awareness set, the insurer does not make positive profit from those contracts according to the insuree. The equilibrium definition requires that the insuree rejects contracts if the corresponding set of compatible beliefs is empty. Under this definition, no equilibrium contract can induce empty set of compatible beliefs.

Theorem 1 *There always exists an equilibrium where the contract is written only on the contingencies that the insuree is initially aware of.*

The idea of the proof goes as follows: The contracts that lead to empty set of compatible beliefs are rejected, therefore any probability distribution can be equilibrium belief corresponding to them. For a contract that corresponds to a non-empty set of compatible beliefs, set the belief so that either the insuree rejects the offer or if she accepts it, then it is not beneficial for the insurer to offer this contract rather than the contract suggested by the theorem. By this belief construction, in equilibrium the insurer offers a contract on S' and provides full insurance on the elements of S' and sets the premium at the level which makes the insuree indifferent between buying or rejecting this offer.

The definition of equilibrium puts minimum restriction on the belief held after each contract. It only requires it to be compatible whenever it is possible. The insuree knows that the insurer is an expected utility maximizer. When a contract is offered in an equilibrium, the insuree may ask herself if this is the best offer for the insurer. One can easily construct examples where some equilibria do not have this property.

The refinement introduced below eliminates this kind of equilibria. It imposes that with respect to the belief held by the insuree, the equilibrium contract should be the best one for the insurer among all the contracts that the insuree can think of. After hearing the equilibrium offer, the insuree can consider only the contracts that would extend her awareness less than the equilibrium contract.

Definition 5 *An equilibrium* $(C^* = (t^*, A^*, k^*),\ D^* : \mathbb{C} \rightarrow \{buy, reject\},\ (P^*)_{C\in\mathbb{C}})$ *is consistent if* $\forall C = (t, A, k) \in \mathbb{C}$ *such that* $A \cup S' \subseteq A^* \cup S'$

$$EU_1^0(C^*, D^*(C^*)|P^*_{C^*}) \geq EU_1^0(C, D^*(C)|P^*_{C^*}).$$

Corollary 1 *There always exists a consistent equilibrium where the signed contract is incomplete.*

This corollary is an immediate implication of Theorem 1 because the theorem states that signing a contract only on S' is always a part of some equilibrium.

In our setting, incompleteness in the contractual form arises as a result of a strategic decision process. Although both complete and incomplete contracts are feasible, the incomplete ones are always signed in some equilibria. There are some examples where all the consistent equilibrium contracts are incomplete.

4 Contracts with Knightian Uncertainty

The equilibrium concept introduced in the previous section is based on the idea that the insuree's equilibrium belief after each contract supports the behavior of the insurer. Generally, each offer induces more than one compatible belief and the beliefs held in an equilibrium in the sense of Section 3 can be any of them. In this section, we impose more structure on the belief formation. We restrict the attention to a type of insuree who does not pick an arbitrary belief from the set of compatible beliefs but instead holds multiple beliefs.

If the insuree is unable to assign a single probability to the newly announced contingencies then the type of uncertainty that the insuree considers contains ambiguity. In this section, we suppose that the insuree is uncertainty (risk and ambiguity) averse. The concave utility function u captures the risk aversion component of the uncertainty aversion. We assume that while evaluating a situation, the insuree uses the maxmin expected utility defined on her multiple belief set and this assumption captures the ambiguity aversion (for behavioral axiomatization of the maxmin expected utility model, see Gilboa and Schmeidler, 1989). In short, the maxmin expected utility model says that the insuree evaluates an offer under every possible scenario she can think of in her multiple belief set and considers the one with the smallest expected utility (the most pessimistic one) as the final evaluation of the offer.

Recall the computer insurance example we gave in the introduction. In the example, Dell Inc. was offering insurance on very costly accidental damages but not on logical recovery. This means that the insurance is partially extending the awareness of the insuree by announcing damages with relatively high cost. Moreover, this contract was advertised by appealing to the pessimism of computer buyers. This is a common property in many insurance contracts. Health and pet insurance plans also frequently stress situations where the cost of medical care without insurance is very high. The belief formation and the solution concept that are developed in this section can explain this observation. We show that when the insuree is ambiguity averse, equilibrium contract either partially extends the awareness of the insuree or does not extend it at all.

When a contract extends the awareness of the insuree, in the construction of multiple belief set we will carry the idea of the set of compatible beliefs introduced in Section 3. The set of multiple beliefs collects all those probabilities that are compatible with the offer of the insurer and assign at least α probability to S' where $\alpha \in (0, \mu(S')]$. Previously we required that a compatible belief assigns non zero probability to every contingency in the extended awareness set. Here when we construct the set of multiple beliefs, we disregard this assumption. It is a technical assumption in order to make the multiple belief set closed.

$$\Pi^*_C := \left\{ \begin{array}{c} P \in \Delta(A \cup S') \mid \\ \\ P(.|S') = \mu(.|S'),\ \alpha \leq P(S') \\ \\ \text{and } EU_1^0(C, D(C)|P) \geq 0 \end{array} \right\} \qquad (1)$$

The ambiguity averse insuree evaluates contract $C = (t, A, k)$ which leads to a non-empty Π^*_C by the following formula:

$$EU_0(C, D(C)|\Pi^*_C) := \begin{cases} \min\limits_{P\in\Pi^*_C} [\sum\limits_{s\in A} u(v-s+t(s)-k)P(s) \\ \quad + \sum\limits_{s\in S'\setminus A} u(v-s-k)P(s)] & \text{if } D(C) = buy \\ \\ \min\limits_{P\in\Pi^*_C} \sum\limits_{s\in A\cup S'} u(v-s)P(s) & \text{if } D(C) = rej. \end{cases}$$

Observe that in the above formula, first the expected utility from offer C is calculated with respect to every compatible belief in Π^*_C and then the smallest of them is the final evaluation of the contract.

Definition 6 *An equilibrium under ambiguity aversion is a pair* $(C^*,\ D^* : \mathbb{C} \to \{buy, reject\})$ *such that*

(i) $C^* \in \arg\max\limits_{C\in\mathbb{C}} EU_1(C, D^*(C))$

(ii) For any $C \in \mathbb{C}$,

$$D^*(C) = \begin{cases} buy & \text{if } EU_0(C, buy|\Pi^*_C) \geq EU_0(C, rej.|\Pi^*_C) \text{ and } \Pi^*_C \neq \emptyset \\ \\ reject & \text{otherwise} \end{cases}$$

where Π^*_C *is the multiple belief set defined in Equation (1).*

Next, we fully characterize the equilibrium contract.

Theorem 2 *Depending on* α*, the equilibrium contract under ambiguity aversion is either signed only on* S' *or it announces one extra contingency besides* S'*. It is a full insurance contract on its domain. Moreover, if it announces a contingency, then the utility of the insuree at that contingency is lower than her expected utility on* S' *without any contract.*

An equilibrium offer has one of the two forms stated in Theorem 2. The idea goes as follows: On the one hand, the insurer would like to inform the insuree about the worst possible contingency because by doing so he can benefit from the pessimism of the insuree. On the other hand, the worst contingency is also the most costly one for the insurer when he promises a transfer on it. Therefore, there is a trade-off between gaining over the premium and losing over the high transfer. If there are contingencies that the insuree is not aware of originally and announcing them makes her pessimistic enough to pay a high premium which compensates the extra transfer that the insurer promises on these contingencies, then the insurer would announce the most beneficial one of them. Otherwise, he will not inform the insuree regarding the unforeseen parts of the uncertainty. Mentioning one new contingency in the contract or not depends on α. If α is small, then it means that when a costly contingency is announced, the insuree will think that this contingency is very probable (she assigns $(1-\alpha)$ probability to it). Therefore, she will be willing to pay high enough premium in order to be covered at that contingency. If α is large, then the corresponding premium that the insuree accepts to pay is not high enough to make the insurer announce this contingency.

If the contract offered in an equilibrium under ambiguity aversion does not extend the awareness of the insuree, then it is the best contract for the insurer also from the insuree's perspective (in the sense of consistency). However, if it announces one extra contingency, then in Knightian uncertainty setting, it is not immediate to conclude that this contract is the best for the insurer according to the insuree. First of all in this setting, we need to be precise with what we mean by consistency (the terminology defined in Section 3). In Section 3, the insuree held a single belief after each contract, so the insuree can calculate the insurer's expected utility with respect to this belief unambiguously. Here, the insuree holds multiple beliefs after the equilibrium offer if it extends the insuree's awareness set. We will check if the equilibrium contract is the best offer for the insurer from the insuree's perspective with respect to every belief in the equilibrium multiple belief set.

Definition 7 *An equilibrium contract $C^* = (t^*, A^*, k^*)$ under ambiguity aversion is the best contract for the insurer according to the insuree if for any $C = (t, A, k) \in \mathbb{C}$ such that $A \cup S' \subseteq A^* \cup S'$, for any $P \in \Pi^*_{C^*}$*

$$EU^0_1(C^*, D^*(C^*)|P) \geq EU^0_1(C, D^*(C)|P)$$

where $\Pi^*_{C^*}$ is the set of multiple beliefs corresponding to C^*.

Proposition 1 *If an equilibrium contract under ambiguity aversion extends the insuree's awareness set, then it is the best contract for the insurer according to the insuree if α (and therefore $\mu(S')$) is sufficiently large.*

The above statement is not surprising because if α is sufficiently large then according to the insuree the newly announced contingency is unlikely. Therefore, she can reason that the insurer wanted to promise her transfer on this contingency because in expectation doing this was not very costly for the insurer.

There are two properties of the optimal contract we find in equilibrium under ambiguity aversion: a) it leaves no extra payoff to the insuree compared to the way she evaluates the situation without a contract, and b) if there are at least two contingencies unforeseen by the insuree, it hides some or all of them. The first property is a characteristic that carries over from the standard theory, without any asymmetry in the awareness. However, the second property tells us that the optimal contract is silent on some contingencies.

5 Competition Promotes Awareness

We saw in the previous section that a monopolistic insurer who has superior awareness will possibly sign an incomplete contract with an ambiguity averse insuree. The equilibrium contract under ambiguity aversion is either silent on all the contingencies that the insuree is unaware of, or it mentions at most one extra contingency. In this section we study if the contracts offered by competing insurers reveal more contingencies. The answer is affirmative and competition indeed promotes awareness.

In standard insurance settings where asymmetric awareness is not an issue, symmetric firms compete over premia. They offer a zero profit contract which is beneficial for the insuree. In our setting, when we introduce competition on the insurers' side, there are two dimensions that the insurers can compete over in their offers: premium and awareness of the insuree. A competing insurer can make a counter offer by either changing the premium or by further extending the awareness of the insuree. We see that competition is an instrument under which not only the insuree can get the cheapest offer but also her unawareness can totally disappear.

Assume there are N risk neutral insurers. All of them are aware of S and believe μ. The ambiguity averse insuree (indexed by 0) is only aware of S', and she believes $\mu(.|S')$ as before. The awareness structure between the insurers and the insuree is the same as in the previous sections. The insuree knows that the insurers are symmetric agents. The insurers make simultaneous offers denoted by $C_i = (t_i, A_i, k_i) \in \mathbb{C}$ for $i = 1, ..., N$. Vector $\mathbf{C} = (C_1, ..., C_N)$ is the collection of insurers' offers. The collection of contracts offered by all insurers except insurer i is denoted by $\mathbf{C}_{-i} = (C_1, ..., C_{i-1}, C_{i+1}, ..., C_N)$.

The offers are exclusive and the insuree may accept, at most, one of the offers or may reject all. The decision of the insuree is denoted by a function $D : \underset{i=1,...,N}{\times} \mathbb{C} \rightarrow \{buy_1, ..., buy_N,\ reject\}$.

For $i = 1, ..., N$, given the offers of other insurers, $\mathbf{C}_{-i}$, and the decision function of the insuree, D, the expected utility of insurer i from contract C_i is:

$$EU_i(\mathbf{C}, D(\mathbf{C})) := \begin{cases} k_i - \sum\limits_{s \in A_i} t_i(s)\mu(s) & \text{if } D(\mathbf{C}) = buy_i \\ 0 & \text{otherwise} \end{cases}$$

The expected utility of insurer i according to the insuree with respect to a probability distribution P is given by

$$EU_i^0(\mathbf{C}, D(\mathbf{C})|P) := \begin{cases} k_i - \sum\limits_{s\in A_i} t_i(s)P(s) & \text{if } D(\mathbf{C}) = buy_i \\ 0 & \text{otherwise} \end{cases}$$

When $\mathbf{C} \in \underset{i=1,...,N}{\times} \mathbb{C}$ is offered, the insuree aggregates the information from each contract in $\mathbf{C}$. She constructs her set of beliefs $\Pi^*_{\mathbf{C}} \subseteq \Delta((\underset{i=1,...,N}{\cup} A_i) \cup S')$ similar to the multiple belief set defined in Section 4. Here the multiple belief set is defined by

$$\Pi^*_{\mathbf{C}} := \left\{ \begin{array}{l} P \in \Delta((\underset{i=1,...,N}{\cup} A_i) \cup S') \mid \\ \\ P(.|S') = \mu(.|S'), \alpha \leq P(S'), \\ \\ EU_i^0(\mathbf{C}, buy_i|P) \geq 0 \\ \text{for } i = 1, ..., N \end{array} \right\} \tag{2}$$

The multiple belief set contains all the probability distributions that are compatible with the offer of each insurer and that assign at least α probability to S'. α is the exogenous lower bound that we defined in Section 4.

With respect to the construction of belief set in Equation (2), the expected utility of the ambiguity averse insuree from vector $\mathbf{C}$ is given by:

$$\begin{aligned} EU_0(\mathbf{C}, buy_i|\Pi^*_{\mathbf{C}}) := & \\ \min_{P\in\Pi^*_{\mathbf{C}}} (& \sum_{s\in A_i} u(v - s + t_i(s) - k_i)P(s) \\ + & \sum_{s\in((\underset{\substack{j=1,...,N\\ j\neq i}}{\cup} A_j)\cup S')\setminus A_i} u(v - s - k_i)P(s)) \end{aligned}$$

$$EU_0(\mathbf{C}, reject|\Pi^*_{\mathbf{C}}) := \min_{P\in\Pi^*_{\mathbf{C}}} \sum_{s\in(\underset{i=1,...,N}{\cup} A_i)\cup S'} u(v - s)P(s)$$

Definition 8 *An equilibrium under ambiguity aversion with competition is a pair*

$(\mathbf{C}^*, D^* : \underset{i=1,...,N}{\times} \mathbb{C} \to \{buy_1, ..., buy_N, reject\})$ *such that*

(i) $C_i^* \in \underset{C_i\in\mathbb{C}}{\arg\max} EU_i(C_i, \mathbf{C}^*_{-i}, D^*(C_i, \mathbf{C}^*_{-i}))$ *for* $i = 1, ..., N$*;*

(ii) For any $\mathbf{C} \in \underset{i=1,...,N}{\times} \mathbb{C}$,

$$D^*(\mathbf{C}) = \begin{cases} buy_i & \textit{if } EU_0(\mathbf{C}, buy_i|\Pi^*_{\mathbf{C}}) \\ & \geq EU_0(\mathbf{C}, buy_j|\Pi^*_{\mathbf{C}}) \\ & \textit{for every } j \neq i \\ \\ & EU_0(\mathbf{C}, buy_i|\Pi^*_{\mathbf{C}}) \\ & \geq EU_0(\mathbf{C}, reject|\Pi^*_{\mathbf{C}}) \\ \\ & \textit{and } \Pi^*_{\mathbf{C}} \neq \emptyset \\ reject & \textit{otherwise} \end{cases}$$

where $\Pi^*_{\mathbf{C}}$ *is defined as in Equation (2).*

We assume that if the insuree is indifferent between two or more offers which are better than rejecting everything, then she picks one of them arbitrarily.

Theorem 3 *There is a symmetric equilibrium under ambiguity aversion with competition where each insurer offers the same zero profit, full insurance, and complete contract, i.e. for any* $i = 1, ..., N$, $C_i^* = (t^*(s) = s, S, k^*)$ *such that* $k^* = \sum_{s\in S} s\mu(s)$.

The equilibrium given in Theorem 3 is a zero profit equilibrium which fully extends the awareness of the insuree. The first step in the proof shows that buying the offer that is suggested by Theorem 3 is better than rejecting all the offers. It is based on the observation that the multiple belief set induced by the equilibrium vector of contracts contains μ.[6] Then we show that no insurer can benefit from deviating to another contract rather than C_i^*.

Recall that according to Definition 8, one of the requirements for the insuree to buy a contract is that the offer should not lead to an empty multiple belief set. Although this assumption did not play a role in determining the equilibrium contract in a single insurer case, it has an impact on the multi-insurer case. Under this requirement, we can easily create examples which have incomplete contracts in equilibrium. Imagine that all the competing insurers offer the same incomplete contract which gives a high expected profit in reality but zero profit according to the insuree. Given the strategies of his opponents, insuree i can deviate to one of the following actions: a) he may announce some contingencies that are hidden by the others' offers, b) he may offer a different contract on the same contingencies. There are examples where non of these deviations is profitable. The idea goes like this: the contingency that is hidden by the other offers is very costly and also very likely. Therefore, the first type of deviation is not profitable enough for insurer i. Alternatively, insurer i can make a different offer on the same set of contingencies as the one announced by his

[6] This argument is based on the assumption that $\mu(S') \geq \alpha$.

opponents (the second type of deviation). This offer may make the multiple belief set empty and it, as a result, may be rejected. Therefore, the best contract for insurer i is the same as the incomplete one offered by all of the other insurers.

We assume that when two or more insurers make the same offer that is good to buy for the insuree, she selects each insurer with an equal probability. For an insurer, the chance of attracting the insuree by offering the same contract as his competitors decreases with the number of insurers making this offer. Therefore, the expected gain of an insurer from following the strategies of the others decreases when the size of competition is large. So if the other insurers are offering an incomplete contract, it may be better for an insurer to further extend the insuree's awareness set . The next result is built on this observation. It shows that if the size of competition is large enough, then the insuree is offered a complete contract in all symmetric equilibria where the insuree accepts an offer.

Theorem 4 *If the number of insurers is large enough, then in any symmetric equilibrium where the insuree buys a contract, the offer is complete, zero profit and full insurance contract.*

6 Discussions

Form of Contracts: We focus on contracts that specify a premium and a transfer rule on the contingencies that the insurer announces. One may suggest two other types of contracts that are excluded in our set of feasible contracts. One of them is the type of contract which, in addition to a premium and a transfer rule, suggests a probability distribution on the contingencies mentioned in the contract. The insurer, who offers the contract, has no incentive to announce the true probability. Therefore, he cannot convince the insuree to believe the suggested distribution. Hence, the insuree would behave as she would without the suggestion.

Another type of contract that one may think of can have a clause such as *anything not specified here is excluded by this contract*. Observe that in our setting the complement of a set of contingencies is not the same set for each agent. Therefore, the statement *anything not specified here* does not refer to the same set of contingencies by the insurer and by the insuree. Indeed, if the contract already mentions everything that the insuree is initially aware of -this is a property of equilibrium contracts we found- then according to the insuree there is nothing excluded in the contract. So, she will not take that clause into account in her evaluation process. Both the contracts which have this clause and the ones without it give the same payoff to the insurer since in our model no transfer takes place if some unspecified contingency is realized. Hence having these contracts in the feasible set would not change the results.

Unawareness: If the insuree is not taking into account some contingencies then we say that she is unaware of them. One may argue that this may not be unawareness because she may have thought about those contingencies and assigned zero probability to them. In this study, we aim to model situations where the insuree may behave differently when some contingencies are announced and when they are not. The insurer does not know the realization of uncertainty when he makes the offer. Then an insuree who already knows the full domain of the uncertainty would not change her belief on this domain after hearing a contract. By calling it unawareness and then allowing it to evolve throughout the interaction, we refer to a new updating rule. Ozbay (2006) discusses in detail why these situations cannot be studied by the tools of standard theory.

Dealing with Knightian Uncertainty: The solution concept introduced in Section 3 is not very restrictive on how the insuree picks her equilibrium belief among all the compatible ones after hearing an offer. In Section 4, we modeled an insuree who cannot hold a single belief but instead considers a subset of compatible beliefs. The evaluation is done by the maxmin expected utility on the multiple belief set.

Alternatively, it may be an interesting exercise to study an insuree who has a distribution $\mathbb{Q}^C$ over the set compatible beliefs corresponding to C. She calculates expected utilities with respect to each belief in the multiple belief set and then computes their mean with respect to $\mathbb{Q}^C$. Maxmin is a type of aggregation rule which puts the highest weight on the worst case scenario in the multiple belief set. It is thus a degenerate distribution on the multiple belief set generated after each contract.

Observe that each specification of $\mathbb{Q}^C$ generates a compatible belief since the set of compatible beliefs is convex and $\mathbb{Q}^C$ is a linear aggregation rule. Hence each correspondence $\mathbb{Q}$ picks one of the equilibria found in Section 3. The equilibrium contract under ambiguity aversion is also one of the contracts signed in equilibria in the sense of Section 3. Consider a sequence of $\{\mathbb{Q}_n\}_{n=1}^{\infty}$ that converges to the aggregation rule $\mathbb{Q}$ that corresponds to this equilibrium. Then the sequence of equilibrium generated by $\{\mathbb{Q}_n\}_{n=1}^{\infty}$ also converges to this equilibrium. Therefore, the equilibrium concept under ambiguity aversion is robust.

7 Conclusion

In this paper, we demonstrate that unawareness can be a basis for contractual incompleteness. We argue that even if complete contracts are also feasible, incomplete contracts emerge for strategic reasons. We study an insurance setting where the set of contingencies that the insurer is aware of is larger than the one the insuree foresees. The insurer can use contracts as a communication device to attract the insuree's attention to the aspects of the uncertainty that the insuree is unaware of. We study different solution concepts to analyze what kind of contracts can be signed in this setting.

Our model is a starting point which relaxes a strong assumption in standard economic theory. It is a realistic exercise to allow for agents who take into account different aspects of an economic situation. Modelling more complicated contractual situations where moral hazard or adverse selection is also an issue would be an insightful research question.

Acknowledgements

I am grateful to Massimiliano Amarante, Pierre-André Chiappori, and Bernard Salanié for their valuable advice. I would like to thank Patrick Bolton, Yeon-Koo Che, Prajit Dutta, Erkut Ozbay, Paolo Siconolfi and seminar participants at Columbia University for their helpful comments and suggestions.

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