Unawareness and Strategic Announcements in Games with Uncertainty

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Abstract

I study games with uncertainty where players have different awareness regarding a chance player's moves (contingencies). An announcer, who is fully aware of the contingencies, can announce some of them to an unaware decision maker (DM) before the DM takes an action. I introduce an equilibrium concept and a refinement to study the way the DM generates her belief on her extended awareness.

1 Introduction

Consider an investor who wants to invest in a stock. During the decision process she may not know all of the contingencies that may affect the stock's price. Moreover, she may not know that she does not know these contingencies, and so on *ad infinitum*. These contingencies do not enter the investor's decision making process. In other words, she is in a state of mind called unawareness. On the other hand, a fully informed stock broker who is advising her, may strategically decide which contingencies to make her aware. After listening to her broker, the investor extends her belief to consider the newly announced contingencies.

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The aim of this paper is to introduce a game theoretical setup to analyze this kind of situations. More specifically, in this paper I will consider games with uncertainty where players might have different awareness regarding the contingencies (the moves of a chance player - nature). There will be two players such that one of them (the announcer) is fully aware while the other player (the decision maker - DM) is unaware of some moves of nature. Furthermore, the announcer knows that the DM is unaware of these moves. However, the DM is not aware that the announcer is aware of some additional moves of nature. In their awareness, both players know the payoffs associated with the moves. Initially, each player has a probability distribution on the moves of nature that they are aware of.

Moreover, the announcer can make an announcement and inform the DM about some moves of nature that were not originally in the DM's state of mind. There is no language barrier between the players. In other words, when some contingencies are announced, the DM can perfectly understand those as relevant contingencies. By perfect understanding, it is meant that the DM comprehends not only the announced moves of nature, but also the payoffs corresponding to those moves. After the announcement, the DM's action and the realization of nature determine the payoffs the players receive.

In this setup, the content of the announcement only includes some or all of the moves of nature. In other words, this is an environment where the announcer can not impose any probability distribution. The key issue in this game is how the DM assigns probabilities to the newly announced contingencies after becoming aware of them. I propose that the belief formation is a part of the equilibrium concept.

Standard game theory assumes that players either know an event or know that they do not know it. Hence, there is no room for unawareness. Two independent studies by Li (2006) and Heifetz, Meier and Schipper (2006) introduced the generalized state space constructions that captured non-trivial unawareness among multi-agents as a generalization of Aumann (1976). Motivated from the possibility of non-trivial unawareness, Copic and Galeotti (2006) and Halpern and Rego (2006) considered extensive form games where players might be unaware of the complete structure of the game (such as unawareness regarding actions of other players). These authors described the game from each player's point of view and introduced a solution concept which was a generalization of Nash equilibrium. Alternatively, Feinberg (2004) epistemically defined unawareness and reasoning about unawareness in dynamic games. He showed that cooperation in a finitely repeated prisoner's dilemma game could be achieved when a player was not fully aware of all the aspects of the game. Feinberg (2005) studied normal form games with incomplete awareness based on the epistemic formulation in Feinberg (2004). He defined the awareness of players regarding the other players' awareness and extended Nash equilibrium as a solution concept.

The aforementioned literature does not allow communication regarding the asymmetric awareness between the players. Players can extend their awareness only if they come to a node in the game that they were not aware of before. If this happens, then the players fully understand those new aspects of the game. However, in this research setting, there is uncertainty about the moves of nature. Once the unaware player becomes aware of some of the contingencies, there is still a problem regarding how she will generate a probability distribution. I propose that while generating a new probability distribution as a part of the equilibrium concept, the unaware player takes into account the fact that the announcement is strategic. That is, the DM thinks that the announcer is expecting higher payoff from his announcement than from another, less informative announcement.

In Section 2, the formal model is introduced. In Section 3, an equilibrium concept for this type of games is defined, and the existence of the equilibrium is studied. A refinement in this setup is suggested. In Section 4, I conclude by discussing why cheap talk with initially assigning zero probability is not satisfactory to capture the equilibria where the DM changes her action after an announcement. Some possible applications of the setup are then provided.

2 The model

There are two players: an announcer and a DM, indexed by 1 and 2, respectively. The set of all moves of nature (contingencies) is a finite set Ω, and the contingencies are distributed by π where $\pi(\omega) \neq 0$ for any $\omega \in \Omega$.

Players have different awareness regarding the contingencies. The announcer is aware of the full set of contingencies, Ω, and believes that the distribution on Ω is π. On the other hand, the DM is only aware of Ω_o, which is a non-empty subset of Ω. The DM believes the conditional distribution of π on Ω_o, $\pi(.|\Omega_o)$. Moreover, the announcer is aware of the DM's limited awareness, while the DM is unaware of the fact that the announcer is of superior awareness. Rather, the DM perceives that the announcer is aware of Ω_o and $\pi(.|\Omega_o)$. The strategy of the announcer is an announcement regarding the moves of nature that the DM is unaware. The announcer does not observe the realization of nature before the announcement. Therefore, $\mathcal{M} := 2^{\Omega \setminus \Omega_o}$ is the set of all strategies of the announcer.

After an announcement, in addition to what she is already aware of (Ω_o), the DM extends her awareness to include all newly announced contingencies. Then, she takes an action from a finite action set, A, which is the same set independent of the announcement. The strategy of the DM is a decision function,

$d : \mathcal{M} \to A$, which specifies the action of the DM after each announcement. The decision function can be thought of as the announcer's belief regarding the behavior of the DM after each announcement; the announcer's belief is confirmed by the action of the DM.

The payoffs received by both players depend on the realization of nature and the action chosen by the DM. Formally, the payoff of player i is determined by utility function $u_i : \Omega \times A \to \mathbb{R}$ for $i = 1, 2$. Observe that utility of the DM is defined on Ω even though she is not aware of Ω initially. When the DM becomes aware of a contingency, she also becomes aware of the associated payoffs.

In order to illustrate the notions introduced so far, an example is constructed:

Example 1 *Let* $\Omega = \{\omega_1, \omega_2\}, \Omega_o = \{\omega_1\}$, *and* $\pi(\omega_1) = \pi(\omega_2) = 0.5$.

The DM has two actions: left, and right. The payoffs are as follows:

		Actions	
		left	*right*
Contingencies	ω_1	*1, 1*	*0, 0*
	ω_2	*0, 0*	*2, 2*

Figure 1 is the game the DM understands initially. Figure 2 is the game that will be understood by the DM when the announcer announces $\{\omega_2\}$.

3 Solution concept

Standard game theory assumes that players' awareness regarding the moves of nature cannot change throughout the game. Relaxing this assumption prevents us from using a solution concept from standard game theory. This issue will be discussed in detail in Section 4.

A solution concept imposes some conditions on an assessment, (M, d, F), which is a triplet containing strategies of each player, M and d, as well as a

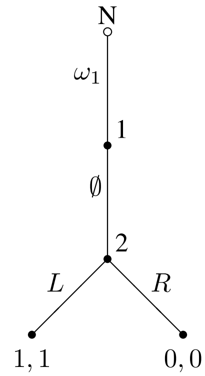

Figure 1: Game that is initially understood by the DM.

belief function F. A belief function, F, assigns to each announcement, M, a probability distribution, F_M, on the union of the sets of contingencies that are announced and the contingencies that are in the initial awareness of the DM, i.e. $M \cup \Omega_o$. The announcer believes that the DM will play based on F, and the belief of the announcer will be confirmed by the DM in the equilibrium.

The deductive interpretation of the solution concepts in standard game theory assumes that each player deduces how the other player will act simply from the rationality principles (see e.g. Osborne and Rubinstein, 1994). Similarly, the solution concept in this paper can be interpreted as the deductive interpretation within the awareness of the players.

Both the announcer and the DM evaluate any strategy by calculating their expected utilities. For any action, $a \in A$, and an announcement, $M \in \mathcal{M}$, the expected utility of the announcer is defined as:

$$EU_1(a) := \sum_{\omega \in \Omega} u_1(\omega, a)\pi(\omega)$$

and the expected utility of the DM, with respect to the probability distribution, F_M, is defined as

$$EU_2(M, a|F_M) := \sum_{\omega \in M \cup \Omega_o} u_2(\omega, a)F_M(\omega)$$

The first requirement, rationality, states that given

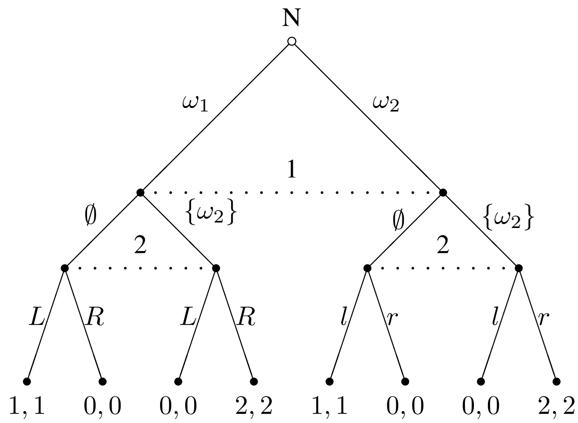

Figure 2: Game that will be understood by the DM when $\{\omega_2\}$ is announced.

the action of the other player and the belief function, both the announcer and the DM respond in order to maximize their expected utilities.

Definition 1 *An assessment* (M^*, d^*, F) *is* ***rational*** *if*

$$M^* \in \underset{M \in \mathcal{M}}{\arg\max} EU_1(d^*(M));$$
$$d^*(M) \in \underset{a \in A}{\arg\max} EU_2(M, a|F_M),$$

for any $M \in \mathcal{M}$.

The DM knows that the announcer does not observe the realization of nature before the announcement. Therefore, an announcement informs the DM about the existence of those contingencies. After hearing an announcement, the DM understands that the announced contingencies are relevant. It is assumed that the announcer cannot announce an impossible contingency, and therefore the DM should believe that any announced contingency is possible.

Definition 2 *A belief function, F, has* ***full support*** *if* $\forall M \in \mathcal{M}$*, and* $\forall \omega \in M \cup \Omega_o$, $F_M(\omega) \neq 0$.

Moreover, it is required that an announcement does not alter the relative weights of the contingencies in the DM's initial awareness. In other words, the conditional of the belief held after each announcement should agree with the initial belief.

Definition 3 *A belief function,* F*,* ***respects the initial belief*** *if*

$\forall M \in \mathcal{M}$*, and* $\forall \omega \in \Omega_o$, $F_M(\omega|\Omega_o) = \pi(\omega|\Omega_o)$.

The DM knows that the announcer is rational. So, the belief of the DM should justify the behavior of the announcer. In other words, after the announcement, the DM forms her belief so that the expected utility of the announcer, according to this belief, is not lower than expected utility of any other announcement that is a subset of the current one. Since the DM can only reason within her revised set of contingencies, she cannot evaluate any other announcement that is not contained in this set.

Definition 4 *An assessment,* (M^*, d^*, F)*, is* ***justifiable*** *if* $\forall M \subseteq M^*$,

$$\sum_{\omega \in M^* \cup \Omega_o} u_1(\omega, d^*(M^*)) F_{M^*}(\omega) \geq \sum_{\omega \in M^* \cup \Omega_o} u_1(\omega, d^*(M)) F_{M^*}(\omega).$$

In the above definition, the left hand side of the inequality corresponds to the announcer's expected utility from the point of view of the DM when M^* is announced. The right hand side is the expected utility of the announcer from the DM's point of view when any other subset of M^* is announced. Note that both of the expected utilities are calculated with respect to F_{M^*}, because while evaluating any subset of M^*, the DM is already aware of M^*. After hearing the announcement, M^*, the DM thinks that the announcer could have announced a subset of M^* but that he preferred to announce M^*. Therefore, the justifiability requires that announcing M^* should be better than announcing any subset of M^* from the point of view of the DM.

Definition 5 *An assessment,* (M^*, d^*, F^*) *is* ***awareness equilibrium*** *if it is rational, justifiable, and* F^* *has full support and respects the initial belief.*

A natural question arising at this point is the existence of assessment that satisfies the equilibrium conditions.

Theorem 1 *Awareness equilibrium always exists.*

Proof. Under no announcement, the DM holds her initial belief, $\pi(.|\Omega_o)$, and set $d^*(\emptyset)$, as a maximal action for the announcer, among the actions that maximize the DM's expected utility.

Define the belief function, F, such that for every announcement, $M \in \mathcal{M}$, F_M respects the initial belief and assigns a small but non-zero probability to all contingencies in the announcement, M, to guarantee that the best response of the DM, $d^*(M)$, is one of the actions that maximizes expected utility of the DM under no announcement. Since A and $\mathcal{M}$ are finite, those probability distributions exist. Since for any $M \in \mathcal{M}$, $d^*(\emptyset)$ is a maximal among all $d^*(M)$ for the announcer, $M^* = \emptyset$. The assessment, $(\emptyset, d^*, F)$, is justifiable since $M^* = \emptyset$. □

In the proof of Theorem 1, under no announcement, the DM takes an action which is a maximal action for the announcer, among the actions that maximize the DM's expected utility. This is a key point for the existence of the equilibrium.

Example 2 *Let* $\Omega = \{\omega_1, \omega_2, \omega_3\}$, $\Omega_o = \{\omega_1\}$, *and* $\pi(\omega_1) = \pi(\omega_2) = \pi(\omega_3) = \frac{1}{3}$.

The DM has two actions: left, and right. The payoffs are as follows:

		Actions	
		left	*right*
	ω_1	$1,1$	$1,1$
Contingencies	ω_2	$1,0$	$0,1$
	ω_3	$0,0$	$2,-4$

The announcer prefers the DM to play "right". The DM is indifferent between "left" and "right" under no announcement. In any equilibrium of this game, $d^*(\emptyset) = right$.

Assume $d^*(\emptyset) = left$.

Clearly, $d^*(\{\omega_2\}) = right$ *and* $d^*(\{\omega_3\}) = left$.

If $d^*(\{\omega_2, \omega_3\}) = left$, *then* $M^* = \{\omega_2\}$ *which is not justifiable.*

If $d^*(\{\omega_2, \omega_3\}) = right$, *then* $M^* = \{\omega_2, \omega_3\}$ *which is not justifiable.*

To see that the announcement $\{\omega_2, \omega_3\}$ *is not justifiable under* $d^*(\{\omega_2, \omega_3\}) = right$, *let* $F_{\{\omega_2,\omega_3\}}(\omega_1) = p$, $F_{\{\omega_2,\omega_3\}}(\omega_2) = q$, $F_{\{\omega_2,\omega_3\}}(\omega_3) = 1 - p - q$ *for* $p, q \in (0, 1)$.

Since $d^*(\{\omega_2, \omega_3\}) = right$, $EU_2(\{\omega_2, \omega_3\}, right|F_{\{\omega_2,\omega_3\}}) \geq EU_2(\{\omega_2, \omega_3\}, left|F_{\{\omega_2,\omega_3\}})$

then $p + q - 4(1 - p - q) \geq p$, $4p + 5q \geq 4$.

Justifiability of $\{\omega_2, \omega_3\}$ *requires* $p + q \leq p + 2(1 - p - q)$, $2p + 3q \leq 2$, *which is a contradiction.*

Hence, for the existence of the equilibrium we must have $d^*(\emptyset) = right$ *which is a maximal action for the announcer among the ones that maximize expected utility of the DM.*

The proof of Theorem 1 suggests that there is always an awareness equilibrium in which the announcer does not make any announcement, and the DM remains within her initial awareness. In those equilibria, the belief is such that the newly announced contingencies are not probable enough to convince the DM to change her action. Such belief function construction may not capture the idea of strategic announcement. Therefore, a refinement that addresses this point is suggested.

In Example 1, at the initial awareness of the DM, she plays "left" which gives some small utility to both players. At the contingency that the DM cannot originally foresee, w_2, playing "right" brings higher utility to both players. The equilibrium suggested by the proof of Theorem 1 makes the DM assign very small probability to this second contingency if she hears about it. Therefore, she keeps playing "left". Since the announcer cannot change the action of the DM by announcing the second contingency, he does not announce it in the equilibrium. In reality, if such a behavior is observed, then one cannot conclude that the DM is irrational. However, if a new contingency is announced, then the DM may reason why this new contingency is announced. When she starts reasoning, she may see that there is an action which improves the payoff of both players if the new announced contingency is probable enough. The DM may think that the announcer wants her to change her action by announcing the new contingency. Since their incentives do not conflict, if there is an extension of awareness, the DM should switch to another action by assigning high enough probability to the newly announced contingency.

Let $\Delta(M \cup \Omega_o)$ be the set of all probability distributions on $M \cup \Omega_o$. Let $\varphi(M, a|d^*)$ be the set of probability distributions with full support on $M \cup \Omega_o$, which make the action $a \in A$ maximal for the DM, respect the initial belief and justify the announcement, $M \in \mathcal{M}$, for a given decision function d^*. Formally,

$$\varphi(M, a|d^*) :=$$

$$\Big\{P \in \Delta(M \cup \Omega_o) : \\ P(\omega) \neq 0 \text{ for any } \omega \in M \cup \Omega_o; \\ P(\omega) = \pi(\omega|\Omega_o) \text{ for any } \omega \in \Omega_o; \\ \sum_{\omega \in M \cup \Omega_o} [u_1(\omega, a)P(\omega) - u_1(\omega, d^*(M'))P(\omega)] > 0 \\ \text{for any } M' \subset M; \\ \text{and } a \in \arg\max_{a' \in A} EU_2(M, a'|P)\Big\}$$

A probability distribution $P \in \varphi(M, a|d^*)$ is called a **reasonable probability distribution** that supports the action $a \in A$, after the announcement $M \in \mathcal{M}$, and $d^* : \mathcal{M} \to A$.

Definition 6 *An awareness equilibrium, (M^*, d^*, F^*) satisfies the **reasoning refinement** if*

for any non-empty $M \in \mathcal{M}$ and $a \in A \setminus \bigcup_{M' \subset M} \{d^*(M')\}$ such that $\varphi(M, a|d^*) \neq \emptyset$, $F^*_M \in \varphi(M, d^*(M)|d^*)$.

The reasoning refinement states that when new contingencies are announced, the DM thinks that the announcer made this announcement to make the DM change her action: After an announcement, $M \in \mathcal{M}$, the DM considers all of the subsets of M and all of the actions she would have taken if these subsets were announced. Then, the DM asks the following questions: Is there an action, $a \in A$, that she would not play if one of the subsets of M was announced? If so, then is there a reasonable probability distribution for M and a? If the DM answers these questions affirmatively, then after the announcement M, she should hold a belief that is a reasonable probability distribution for M and $d^*(M)$. Hence, she plays an action that would not have been played after any subset of M.

In Example 1, under no announcement, the DM plays "left". When $\{\omega_2\}$ is announced, the DM thinks that the announcer wants her to take another action, "right". The DM can support playing "right" by a reasonable probability distribution which assigns ω_2 a probability that is greater than or equal to $\frac{1}{3}$. So, the reasoning refinement disallows assigning a very small probability to ω_2, as the equilibrium in the proof of Theorem 1 suggests. Under this belief, the announcer prefers announcing $\{\omega_2\}$ to making no announcement. Therefore, no announcement is not always part of an awareness equilibrium that satisfies the reasoning refinement.

Theorem 2 *Awareness equilibrium that satisfies reasoning refinement always exists.*

Proof. Let F be the belief function constructed in the proof of Theorem 1.

Under no announcement, the DM holds her initial belief, $\pi(.|\Omega_o)$, and set $d^*(\emptyset)$, as a maximal action for the announcer among the actions that maximize the DM's expected utility.

Construct F^* and d^* inductively:

For $n \in \mathbb{N}$, let for any $M' \in \mathcal{M}$ s.t. $|M'| < n$, $d^*(M')$ and $F^*_{M'}$ be constructed.

Construct F^*_M and $d^*(M)$ for $|M| = n$:

If there is an action, $a \in A$, that would not have been played after any announcement that is a proper subset of M, and if there is a reasonable probability distribution, P, on $M \cup \Omega_o$, supporting the action a after the announcement M, then set $F^*_M = P$ and $d^*(M) = a$.

Otherwise, set $F^*_M = F_M$, and $d^*(M)$ as one of the actions that maximizes expected utility of the DM under no announcement (same as in the proof of Theorem 1).

Given the decision function, d^*, the announcer announces M^*, which maximizes his expected utility.

This construction of the assessment (M^*, d^*, F^*) is an awareness equilibrium satisfying the reasoning refinement. □

The reasoning refinement emphasizes the strategic nature of the announcement. The announcer, who would like the DM to take the right action for the announcer, may prefer to hide some of the contingencies. Even if there is no conflict of interest, in order to correct the DM's action, it is not always optimal to reveal all of the contingencies. For example, consider a child who is not taking the correct action for herself due to her limited awareness. The parent, who has the same preference as his child, would like to advise her in order to direct her to the right action. In such a case, too much advice might confuse the child and lead her to a wrong action. In the example below, there is a strategy-wise unique awareness equilibrium that satisfies reasoning refinement and in this equilibrium all of the contingencies are not announced.

Example 3 *Let* $\Omega = \{\omega_1, \omega_2, \omega_3\}$ *with* $\pi(\omega_1) = 0.1$, $\pi(\omega_2) = 0.8$, $\pi(\omega_3) = 0.1$. *and* $\Omega_o = \{\omega_1\}$.

		Actions		
		left	*middle*	*right*
	ω_1	$3,3$	$0,0$	$2,2$
Contingencies	ω_2	$0,0$	$7,7$	$2,2$
	ω_3	$2,2$	$0,0$	$2,2$

Here, one can easily check that the unique strategies in any awareness equilibrium satisfying reasoning refinement are as follows: The announcer announces only ω_2; *the DM plays "left" under no announcement, "middle" when* ω_2 *is announced, "left" when* ω_3 *is announced, and "right" when both* ω_2 *and* ω_3 *are announced.*

In the above example, the announcer would like the DM to take "middle" action. Within the initial awareness, the DM plays "left". If the announcer announces ω_2, then the DM reasons that the announcer wants her to change her action. Under reasoning refinement, the DM concludes that with respect to a certain belief switching to "middle" improves both the announcer's and the DM's payoffs. When ω_3 is announced, there is no such a belief. So, she keeps playing "left". Therefore, announcing only ω_2 is enough to correct the action of the DM. However, when all of the contingencies are announced, the DM reasons that the announcer wants her to take an action different than "left" and "middle". She thinks that if the announcer wanted her to play "left" or "middle", he could do it by announcing less. So, the DM seeks for another action and a belief such that playing that action is better for both the DM and the announcer from the DM's point of view, and "right" is that action. Hence, the announcer does not want to announce everything in order to prevent the DM to make the wrong reasoning.

4 Concluding Remarks

In this paper, we study games with uncertainty where players have different awareness regarding the moves of nature. The fully aware player decides which contingencies to announce to the less aware player. The key element in this game is the belief formation procedure, which is based on the fact that "the announcement is a strategic decision of the rational announcer". Additionally, I introduce a refinement concept that further emphasizes the strategic nature of the announcement. In a simple example, I show that even if there is no conflict of interest, it is not always optimal to announce all of the contingencies. Announcing too many contingencies can lead the DM to make wrong inferences.

A standard solution concept cannot be applied immediately to this setup, because in standard game theory the awareness of the players is fixed throughout the game. Nevertheless, one may reformulate this setup as the DM is aware of all the contingencies, but assigns zero probability to some of them. The announcer does not know the realization of nature, but he knows the true distribution of nature. Then, a type space can be defined on the distribution of nature. The DM does not know the true type of the announcer and holds a degenerate prior, which puts a probability of 1 to the type that the DM initially believes. Then, the announcement can be thought of as cheap talk (see Crawford and Sobel, 1982). Since the DM's prior belief is degenerate, the posterior belief will be the same as the prior after each announcement. So, in a perfect Bayesian equilibrium, the announcement cannot be informative and the DM will not change her action after an announcement. Hence, the standard tools will not explain the instances when the DM wants to take a different action after an announcement.

A natural application of this setup is in contractual environments. Since A contract can be thought of as a communication device that extends the awareness of the contracting parties, the equilibrium concept introduced here can be used. Indeed, Filiz (2006) considered this model and solution concept in a contract setting. She showed that although it is feasible to sign a complete contract, in equilibrium an incomplete contract is signed. Moreover, she extended the setup by allowing the unaware agent to hold multiple beliefs after an announcement. She demonstrated that if the unaware agent is ambiguity averse, then the equilibrium contract is unique and incomplete. Furthermore, by allowing more than one announcer (insurer in her setup), she showed that competition among the announcers extends the awareness of the DM (insuree in her setup).

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