BELIEF CHANGE AND DEPENDENCE

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Abstract

It is a very natural requirement for belief change operations that formulas that are independent of a given update should be preserved. Such a proposal has already been made by Gärdenfors (1990). In this perspective we study the links between belief change and the notion of dependence, our aim being to give dependence axioms able to characterize the AGM postulates.

Proceeding exactly in the same way as Gärdenfors (1988) did in the case of epistemic entrenchment, we show how a given dependence relation can be used to define a contraction operation, and the other way round we show how an independence relation can be obtained from a given contraction operation. The grande finale is a characterization theorem.

1 INTRODUCTION

The central question in the theory of belief revision is what is meant by a minimal change of a state of belief. The well-known AGM rationality postulates for contraction (Alchourrón et al. 1985, Gärdenfors 1988) partially answer that question:

(K^-1) $K - A$ is a theory.

(K^-2) $K - A \subseteq K$.

(K$^-$3) If $A \notin K$ then $K \subseteq K - A$.

(K$^-$4) If $A \in K - A$ then $\vdash A$.

(K$^-$5) $K \subseteq (K - A) + A$ (where $+$ is expansion). [1]

(K$^-$6) If $A \leftrightarrow B$ then $K - A = K - B$.

(K$^-$7) If $C \in K - A$ and $C \in K - B$ then $C \in K - A \wedge B$.

(K$^-$8) If $A \notin K - A \wedge B$ then $K - A \wedge B \subseteq K - A$.

They do that by giving requirements for the interplay between the belief change operator $-$ on the one hand and the classical connectives $\wedge$, $\vee$, $\neg$ and the notions of consistency and theoremhood on the other.

As pointed out in Gärdenfors (1990), "the criteria of minimality that have been used [in the models for belief change] have been based on almost exclusively logical considerations. However, there are a number of non-logical factors that should be important when characterizing a process of belief revision." Gärdenfors focusses the notion of dependence (he uses the synonymous term 'relevance') and proposes the following preservation criterion:

If a belief state is revised by a sentence A,
then all sentences in K that are independent of the validity of A
should be retained in the revised state of belief.

This seems to be a very natural requirement for belief revision operations. But as Gärdenfors notes, "a criterion of this kind cannot be given a technical formulation in a model based on belief sets built up from sentences in a simple propositional language because the notion of relevance is not available in such a language."

Our aim is to give a formal account of the notion of dependence, and to employ it in belief change. More precisely, our question is whether the AGM-postulates have a natural counterpart in terms of dependence (just as they have one in terms of epistemic entrenchment).

In this spirit, we have already investigated the notion of dependence in possibility theory (Fariñas and Herzig 1994a, Dubois et al. 1994)), which is

[1] Expansion is defined by $K + A = \{C : K \cup \{A\} \vdash C\}$.

an uncertainty theory that is very close to epistemic entrenchment (Dubois and Prade 1991, 1992).

Dependence can be based as well on the notion of theme, supposing that to each formula there is associated a set of themes (Fariñas and Lugardon 1991, Lugardon 1996). The themes of a formula are what the formula is about. Then one defines two formulas to be dependent if they have some theme in common. (Such a dependence is called conversational in (Cohen 1994)).

In the same way as in this paper, a dependence-based preservation criterion has been used in the fields of nonmonotonic reasoning (Delgrande and Pelletier 1994, Benferhat et al. 1994), knowledge base updates (Fariñas and Herzig 1994b, 1994c, 1996), and reasoning about actions (Gasquet and Herzig 1995).

2 BACKGROUND: PROBABILISTIC INDEPENDENCE

There are two different notions of independence that can be defined in probability theory, viz. independence between variables and independence between events. The former has been studied extensively e.g. in (Pearl 1988, Studený 1993, Spohn 1994). In this paper we investigate independence between events, which in a logical language corresponds to independence between formulas.

2.1 The Multiplication Law

Traditionally, the formal basis of the dependence relation (that has also been called relevance relation) is probability theory. There, the standard definition of probabilistic independence [2] is via the so-called Multiplication Law:

$$B \text{ is independent of } A \text{ iff } Pr(A \wedge B) = Pr(A) \times Pr(B)$$

[2]It seems to be more natural to define independence as 'B is probabilistically independent of A iff $Pr(B|A) = Pr(B)$', relying thus on conditioning, but this is equivalent to the Multiplication Law.

It follows from the axioms of probability theory that independence defined in this way is a symmetric relation, and that it is not sensitive to negation. In other words:

- If B is independent of A then A is independent of B (symmetry).
- If B is independent of A then B is independent of $\neg A$ (negation).

Symmetry justifies to say "A and B are independent" instead of "A is independent of B". Moreover we have

- If $\vdash A \leftrightarrow B$ then A and C are independent iff B and C are independent (equivalence).
- A and $\top$ are independent (tautology).

Note that there are no such simple properties governing the interplay of independence with conjunction and disjunction.

The above properties are not enough to completely characterize the notion of probabilistic independence. We need two more axioms ((Kolmogorov 1956), cited in (Fine 1973)):

- If A and B are independent, A and C are independent and $\vdash \neg(B \wedge C)$ then A and $B \vee C$ are independent.
- If A and B are independent, C and D are independent, $B \geq D$, and $A \geq C$ then $A \wedge B \geq C \wedge D$.

In this way the axiomatisation of dependence involves not only logical truth, but also a qualitative probability relation "$\geq$". Hence we cannot study the formal properties of such a dependence notion separately from the probabilistic framework.

The Multiplication Law has been criticized by several authors.

R. von Mises (1963) has argued that A and B could be independent by the Multiplication Law just because of "pure numerical accidents", although A and B are not intuitively independent in the sense of 'being separated' or 'not influencing each other'. He gives an example where $Pr(A) = Pr(B) = Pr(C) = Pr(D) = 1/4$, and A, B, C and D are mutually exclusive (i.e. $\vdash \neg(A \wedge B), \vdash \neg(A \wedge C), \vdash \neg(A \wedge D), \vdash \neg(B \wedge C), \ldots$). Then he investigates whether $A \vee B$ and $B \vee C$ are independent. According to von Mises they are intuitively dependent (because they have B in common), whereas the Multiplication Law says that they are independent.

In the rest of the section we present two important formal principles that are not validated by the Multiplication Law.

2.2 The Conjunction Criterion for Dependence

A formal objection to the multiplication Law has been given by J. M. Keynes (1921, cited in (Gärdenfors 1978)). According to Keynes, the following conjunction criterion for dependence should be valid:

(CCD) If C depends on A, and C depends on B then C depends on $A \wedge B$.

He notes that (CCD) is not validated by the Multiplication Law.

Keynes proposes a stronger definition of probabilistic independence that does it:

$$\begin{gathered} C \text{ is independent of } A \text{ iff} \\ \text{there is no } B \text{ such that } \vdash A \rightarrow B \text{ and } Pr(C|B) \neq Pr(C). \end{gathered}$$

R. Carnap (1950) has shown that this definition leads to a trivial notion of independence: It entails that C depends on A as soon as C and A are consistent.

2.3 The Conjunction Criterion for Independence

P. Gärdenfors (1978) has suggested that not only (CCD), but also its dual

(CCI) If C is independent of A, and C is independent of B then C is independent of $A \wedge B$.

is a natural principle that should be valid. He criticizes the Multiplication Law because it does not guarantee (CCI).

He shows that in the probabilistic framework, adding both principles leads to trivialization. Gärdenfors prefers dropping (CCD), and he investigates a series of stronger probabilistic definitions of independence. He finally comes up with

$$\begin{gathered} C \text{ is independent of } A \text{ iff} \\ Pr(A) = 0, \text{ or } Pr(C|A \wedge B) = Pr(C) \text{ for all } B \text{ such that } Pr(A \wedge B) > 0 \\ \text{and } Pr(C|B) = Pr(C) \end{gathered}$$

This definition validates (CCI).

In a subsequent paper (1990), Gärdenfors investigates the link between probabilistic independence and revision. He proposes a new definition in terms of conditional probability and contraction of probability measures, and he gives a set of properties of dependence relations. He proves that for every contraction function these properties are satisfied. It remains open whether these properties exactly characterize contraction operations.

2.4 The Rest of the Paper

In section 3 we axiomatize dependence relations. In section 4 we show how a given dependence relation can be used to define a contraction operation, and the other way round in section 5 we show how a dependence relation can be obtained from a given contraction operation. Finally in section 6 we establish several characterization theorems.

3 AXIOMATICS

Formally, what we investigate is a binary relation on formulas that we call dependence relation and that we note $\rightsquigarrow$. $A \rightsquigarrow B$ is read "B depends on A". The complement of $\rightsquigarrow$ is noted $\not\rightsquigarrow$, and $A \not\rightsquigarrow B$ is read "B is independent of A".

Throughout the paper we suppose a language of classical propositional logic, with conjunction $\wedge$, disjunction $\vee$, implication $\rightarrow$, equivalence $\leftrightarrow$, and negation $\neg$. $\top$ and $\bot$ are propositional constants denoting truth and falsehood respectively. $A, B, C \ldots$ denote formulas. $\vdash$ is the classical consequence relation.

Given a theory K, we say that a relation $\rightsquigarrow$ on formulas is a dependence relation if it satisfies:

(LE^l) If $A \leftrightarrow B$ and $A \rightsquigarrow C$ then $B \rightsquigarrow C$.

(LE^r) If $B \leftrightarrow C$ and $A \rightsquigarrow B$ then $A \rightsquigarrow C$.

(CCI^l) If $A \wedge B \rightsquigarrow C$ then $A \rightsquigarrow C$ or $B \rightsquigarrow C$.

(CCI^r) If $A \rightsquigarrow B \wedge C$ then $A \rightsquigarrow B$ or $A \rightsquigarrow C$.

(Def-K) $A \in K$ iff either $\vdash A$ or there is some C such that $A \rightsquigarrow C$.

(Cond-ID) If $A \rightsquigarrow C$, then $A \rightsquigarrow A$.

(Disj) If $\vdash A \vee C$ then $A \not\rightsquigarrow C$.

($\text{CCD}_0{}^r$) If $A \rightsquigarrow B$ and $C \rightsquigarrow C$ then $A \rightsquigarrow B \wedge C$.

($\text{CCD}_0{}^l$) If $A \rightsquigarrow C$ and $A \wedge B \rightsquigarrow A$, then $A \wedge B \rightsquigarrow C$.

Remarks. (LE^l) and (LE^r) are the standard axioms of syntax independence.

(CCI^l) is the 'conjunction criterion for independence' studied by Gärdenfors (1990).

(CCI^r) is the symmetric counterpart of (CCI^l), which we must give here because $\rightsquigarrow$ is not necessarily symmetric.

(Def-K) establishes a link between the theory and the dependence relation. If $A \in K$ and $\not\vdash A$ then K is effectively contractable by A, in the sense that there are formulas C such that when we contract by A then C is thrown out. Hence $K - A$ is different from K.

(Cond-ID) means that if there is something depending on A then A depends on itself. This permits to read $A \rightsquigarrow A$ as 'A is a contingent truth', i.e. $A \in K$ and $\not\vdash A$. A being a contingent truth means two things: first, contraction by A throws out some formula C. (Conversely if A is not a contingent truth then $K - A = K$.) Second, there is some formula B such that contraction by B throws out A. Formally, $A \rightsquigarrow A$ will be equivalent to both 'there is B such that $B \rightsquigarrow A$', and to 'there is C such that $A \rightsquigarrow C$'.

(Disj) involves negation. It is the exact counterpart of the recovery postulate ($\text{K}^{-}5$). An equivalent formulation is that $A \not\rightsquigarrow \neg A \vee C$. It follows from it that $A \not\rightsquigarrow \neg A$, i.e. A and $\neg A$ are always independent. This contrasts with the probabilistic independence notion, where A and $\neg A$ are always dependent. It follows as well that $\top \not\rightsquigarrow C$, $A \not\rightsquigarrow \top$, and $\bot \not\rightsquigarrow \top$.

($\text{CCD}_0{}^r$) is close to Keynes's (CCD) (and in fact somewhat stronger).

($\text{CCD}_0{}^l$) is the counterpart of the AGM-postulate ($\text{K}^{-}8$).

In fact, a given dependence relation defines a theory

$$K_{\rightsquigarrow} = \{A : \vdash A \text{ or there is some } C \text{ such that } A \rightsquigarrow C\}$$

By (Cond-ID), we can simplify to $K_{\rightsquigarrow} = \{A : \vdash A \text{ or } A \rightsquigarrow A\}$. (Note that this is very similar to epistemic entrenchment based belief change, where $K_{\leq} = \{A : \text{there is some } B \text{ such that } B < A\} = \{A : \neg A < A\}$.)

Keynes's 'conjunction criterion for dependence' (CCD) does not appear in our list, but it will nevertheless be a valid principle (see section 5).

4 FROM DEPENDENCE TO CONTRACTION

Given a dependence relation $\rightsquigarrow$, just as in the case of epistemic entrenchment we can define a contraction operation via the following condition.

(Cond -). $C \in (K_{\rightsquigarrow} - A)$ iff either $\vdash C$ or $C \rightsquigarrow C$ and $A \not\rightsquigarrow C$.

In this section we shall drop the index and write K for $K_{\rightsquigarrow}$.

Remark. In presence of (Def-K), an equivalent condition is $C \in K - A$ iff $C \in K$ and $A \not\rightsquigarrow C$. This expresses Gärdenfors's requirement that beliefs that are independent of A should 'survive' a contraction by A.

We can prove that $-$ is an AGM-contraction:

Theorem 1 *Given two relations $\rightsquigarrow$ and $-$ such that (Cond $-$) holds, if $\rightsquigarrow$ is a dependence relation then $-$ is an AGM contraction (for the theory $K_{\rightsquigarrow}$).*

Proof. We check the postulates one by one.

($K^{-}1$) $K - A$ is a theory.

We must prove that $K - A$ is closed under implications. Let $B \in K - A$ and $B \rightarrow C \in K - A$. By the definition of $-$, $A \not\rightsquigarrow B$ and $A \not\rightsquigarrow B \rightarrow C$, as well as $B \rightsquigarrow B$. By (CCI^r), $A \not\rightsquigarrow B \wedge (B \rightarrow C)$. By (LE^r), $A \not\rightsquigarrow B \wedge C$. By (CCD$_0{}^r$), $A \not\rightsquigarrow C$ or $B \not\rightsquigarrow B$. If $\vdash B$ then $B \wedge C$ is equivalent to C, and by (LE^r) $A \not\rightsquigarrow C$, entailing by the definition of $-$ that $C \in K - A$. Else suppose that $\nvdash B$. As $B \in K$, by (Def-K) there is some C such that $B \rightsquigarrow C$, and we get $B \rightsquigarrow B$ by (Cond-ID). Hence we must have $A \not\rightsquigarrow C$, and as $C \in K$, by the definition of $-$ we have $C \in K - A$.

(K^-2) $K - A \subseteq K$.

Suppose $C \in K - A$. Then either $\vdash C$ or $C \rightsquigarrow C$ and $A \not\rightsquigarrow C$. In the former case we have $C \in K$. In the latter case, $C \in K$ follows from $C \rightsquigarrow C$ by (Def-K).

(K^-3) If $A \in K$ then $K \subseteq K - A$.

Let $A \notin K$. By (Def-K), $A \not\rightsquigarrow C$ for every C. Suppose $C \in K$. By (Def-K) and (Cond-ID), either $\vdash C$ or $C \rightsquigarrow C$. Both imply $C \in K - A$ by the definition of $-$.

(K^-4) If $A \in K - A$ then $\vdash A$.

Let $A \in K - A$. By definition we have either $\vdash A$ or $A \rightsquigarrow A$ and $A \not\rightsquigarrow A$. Hence $\vdash A$ by (Def-K).

(K^-5) $K \subseteq (K - A) + A$ (where $+$ is expansion).

An equivalent formulation is "If $C \in K$ then $\neg A \vee C \in K - A$". Let $C \in K$. Hence $\neg A \vee C \in K$, and $\neg A \vee C \rightsquigarrow \neg A \vee C$ by (Def-K). As by (Disj), $A \not\rightsquigarrow \neg A \vee C$, we have $\neg A \vee C \in K - A$ by the definition of $-$.

(K^-6) If $A \leftrightarrow B$ then $K - A = K - B$.

Let $C \in K - A$. By the definition of $-$, $A \not\rightsquigarrow C$. By (LE^l), $B \not\rightsquigarrow C$. As $C \in K$, again by the definition of $-$ we must have $C \in K - B$, too.

(K^-7) If $C \in K - A$ and $C \in K - B$ then $C \in K - A \wedge B$.

By the definition of $-$ we have $A \not\rightsquigarrow C$ and $B \not\rightsquigarrow C$, and either $\vdash C$ or $C \rightsquigarrow C$. The former two entail $A \wedge B \not\rightsquigarrow C$ by (CCI^l). Again by the definition of $-$, both cases of the latter entail $C \in K - A \wedge B$.

(K^-8) If $A \notin K - A \wedge B$ then $K - A \wedge B \subseteq K - A$.

We consider two cases: If $A \notin K$ then by (K^-3), $K \subseteq K - A$, and as $K - A \wedge B \subseteq K$ by (K^-2), we get $K - A \wedge B \subseteq K - A$. (($K^-3$) and ($K^-2$) have already been established.) If $A \in K$ then $A \wedge B \rightsquigarrow A$ by the definition of $-$. Let $C \in K - A \wedge B$. Hence $A \wedge B \not\rightsquigarrow C$. By ($CCD_0{}^l$), $A \not\rightsquigarrow C$, and hence $C \in K - A$.

5 FROM CONTRACTION TO DEPENDENCE

Given a theory K and a contraction operation $-$, we are able to define $\leadsto$ via the following condition.

$$\textbf{(Cond } \leadsto \textbf{)}. \; A \leadsto C \text{ iff } C \in K \text{ and } C \notin K - A.$$

In this way we can define a relation $\leadsto$ from a given AGM contraction operation. We can prove that $\leadsto$ is a dependence relation:

Theorem 2 *Given two relations $\leadsto$ and $-$ such that (Cond $\leadsto$) holds, if $-$ is an AGM contraction, then $\leadsto$ is a dependence relation.*

Proof. We check the axioms for dependence relations.

(LE^r) If $B \leftrightarrow C$ and $A \leadsto B$ then $A \leadsto C$.

This follows from (K^-1).

(LE^l) If $A \leftrightarrow B$ and $A \leadsto C$ then $B \leadsto C$.

This follows from (K^-6).

(CCI^r) If $A \leadsto B \wedge C$ then $A \leadsto B$ or $A \leadsto C$.

Let $A \leadsto B \wedge C$, i.e. $B \wedge C \in K$ and $B \wedge C \notin K - A$. By (K^-1), $K - A$ is a theory, and therefore $B \notin K - A$ or $C \notin K - A$. Hence $A \leadsto B$ or $A \leadsto C$ by the definition of $\leadsto$.

(CCI^l) If $A \wedge B \leadsto C$ then $A \leadsto C$ or $B \leadsto C$.

Let $A \wedge B \leadsto C$, i.e. $C \in K$ and $C \notin K - A \wedge B$. By (K^-7), $C \notin K - A$ or $C \notin K - B$. Consequently, $A \leadsto C$ or $B \leadsto C$ by the definition of $\leadsto$.

(Def-K) $A \in K$ iff either $\vdash A$ or there is some C such that $A \leadsto C$.

From the left to the right, we check two cases. First, if $A \notin K - A$ then $A \leadsto A$ by (Cond $\leadsto$) and we are done. Else if $A \in K - A$ then $\vdash A$ by (K^-4).

From the right to the left, if $\vdash A$ then we are done. If there is some C such that $A \leadsto C$, then $C \in K$ and $C \notin K - A$ by the definition of $\leadsto$. Hence K is different from $K - A$, and consequently $A \in K$ by (K^-3).

(Cond-ID) If $A \rightsquigarrow C$, then $A \rightsquigarrow A$.

Let $A \rightsquigarrow C$. By the definition of $\rightsquigarrow$, $A \in K$ and $K \not\subseteq K - A$. Suppose $A \in K - A$. Then $\vdash A$ by ($K^{-}4$), and by the recovery postulate ($K^{-}5$) we would have $K \subseteq K - A$, which is contradictory.

(Disj) If $\vdash A \vee C$ then $A \not\rightsquigarrow C$.

Let $\neg A \vee B \in K$. We have two cases: If $A \notin K$ then $K \subseteq K - A$ by ($K^{-}3$) and we are done. If $A \in K$ then by ($K^{-}1$) we must have $B \in K$, too. Now by the recovery postulate ($K^{-}5$), $\neg A \vee B \in K - A$.

($CCD_0{}^r$) If $A \rightsquigarrow B$ and $C \rightsquigarrow C$ then $A \rightsquigarrow B \wedge C$.

Let $A \rightsquigarrow B$ and $C \rightsquigarrow C$, i.e. $B \in K$, $B \notin K - A$, and $C \in K$. Hence $B \wedge C \in K$. If we suppose $B \wedge C \in K - A$, then by ($K^{-}1$) we would have $B \in K - A$, which is contradictory.

($CCD_0{}^l$) If $A \rightsquigarrow C$ and $A \wedge B \rightsquigarrow A$, then $A \wedge B \rightsquigarrow C$.

Let $A \rightsquigarrow C$ and $A \wedge B \rightsquigarrow A$, i.e. $C \in K$, $A \in K$, and $C \notin K - A$ and $A \notin K - A \wedge B$. By ($K^{-}8$), $C \notin K - A \wedge B$, hence $A \wedge B \rightsquigarrow C$.

Remark. Note that both versions of Keynes's conjunction criterion for depedence

(CCD^r) If $A \rightsquigarrow B$ and $A \rightsquigarrow C$ then $A \rightsquigarrow B \wedge C$.

(CCD^l) If $A \rightsquigarrow C$ and $B \rightsquigarrow C$, then $A \wedge B \rightsquigarrow C$.

are valid, in the sense that whenever $-$ is an AGM contraction and (Cond $\rightsquigarrow$) holds, then $\rightsquigarrow$ satisfies (CCD^r) and (CCD^l).

6 CHARACTERIZATION THEOREMS

In the preceding sections we have shown how to obtain AGM contractions from dependence relations and vice versa. The whole does not give us a characterization theorem yet, because we have nothing said about the relation between the conditions (Cond $\rightsquigarrow$) and (Cond $-$). This is what we are going to do now.

The result we would like to have is that for every given relation $\rightsquigarrow$ and operation $-$ verifying (Cond $\rightsquigarrow$), $\rightsquigarrow$ is a dependence relation iff $-$ is an AGM contraction. Unfortunately, it turns out that (Cond $\rightsquigarrow$) and the axioms for $\rightsquigarrow$ are not able to guarantee ($K^{-}2$). The reason is that for a given A, the set of C such that $A \rightsquigarrow C$ is exactly the set difference of K and $K - A$. Therefore, $\rightsquigarrow$ does not give us any information on the set difference of $K - A$ and K (and we ignore thus whether $K - A$ is included in K). Therefore we must explicitly assume that $-$ is not any belief change operation, but what we call a *subtraction operation*, i.e. an operation satisfying $K - A \subseteq K$.

Theorem 3 *Let $\rightsquigarrow$ be any relation on formulas. Let $-$ be any subtraction operation. Let (Cond $\rightsquigarrow$) hold: $A \rightsquigarrow C$ iff $C \in K$ and $C \notin K - A$. Then $-$ is an AGM contraction iff $\rightsquigarrow$ is a dependence relation.*

Proof. From the left to the right, the proof is done by the "from contraction to dependence" theorem.

From the right to the left, let $\rightsquigarrow$ be a dependence relation and let (Cond $\rightsquigarrow$) hold. We can apply the "from dependence to contraction" theorem if we can show that (Cond $-$) holds, i.e. $C \in K - A$ iff either $\vdash C$ or $C \rightsquigarrow C$ and $A \not\rightsquigarrow C$. Let us suppose first that $C \in K - A$. By (Cond $\rightsquigarrow$), $A \not\rightsquigarrow C$. As by hypothesis $-$ is a subtraction operation, we have $C \in K$. In the other sense, suppose $C \notin K - A$. By (Cond $\rightsquigarrow$), $C \notin K$ or $A \rightsquigarrow C$.

Remark. It is also possible to state a characterization theorem based on (Cond $-$). In this case, in order to prove that (Cond $\rightsquigarrow$) holds we must presuppose that $\rightsquigarrow$ is not an arbitrary relation on formulas, but one satisfying that $A \rightsquigarrow C$ implies $C \in K$.

In the rest of the section we give characterize weaker dependence relations. In particular the basic postulates for contraction ($K^{-}1$) - ($K^{-}6$) are characterized.

Theorem 4 *Let $\rightsquigarrow$ be any relation on formulas. Let $-$ be any subtraction operation. Let (Cond $\rightsquigarrow$) hold.*

1. *$-$ satisfies ($K^{-}1$) - ($K^{-}6$) iff $\rightsquigarrow$ satisfies (LE^{l}), (LE^{r}), (CCI^{r}), (Def-K), (Cond-ID), (Disj), ($CCD_0{}^{r}$).*
2. *$-$ satisfies ($K^{-}2$), ($K^{-}4$), ($K^{-}6$), ($K^{-}7$) iff $\rightsquigarrow$ satisfies (LE^{l}), (CCI^{l}), (Def-K).*

Proof. The proof can be done by examining the relevant cases in the proofs of the 'from dependence to contraction' and 'from contraction to dependence' theorems. Both are put together in the same way as in the preceding characterization theorem.

7 CONCLUSION

We have studied the notion of dependence and its relation with belief change operations. We have established a strong link between contraction and dependence. In particular the following nice principles are all valid:

(CCI^r) If $A \leadsto B \wedge C$ then $A \leadsto B$ or $A \leadsto C$.

(CCI^l) If $A \wedge B \leadsto C$ then $A \leadsto C$ or $B \leadsto C$.

(CCD^r) If $A \leadsto B$ and $A \leadsto C$ then $A \leadsto B \wedge C$.

(CCD^l) If $A \leadsto C$ and $B \leadsto C$, then $A \wedge B \leadsto C$.

Hence we meet both Keynes's conjunction criterion for dependence (CCD) and Gärdenfors's conjunction criterion for independence (CCI).

With the Levi identity it is possible to obtain similar results for belief revision. One can as well obtain similar results for nonmonotonic reasoning, using the correspondence of (Makinson and Gärdenfors 1990) between the former and revision.

We are convinced that dependence is a fruitful notion in the study of belief change and nonmonotonic reasoning, and that a lot of things remain to be done in this area. In particular we think that it will be a useful tool in the practical implementation of contraction and revision operations.

H. Reichenbach (1949) has argued that dependence and independence should be ternary rather than binary relations, where the third element in the relation is the evidence on which we declare that A and C are independent. From the ternary relation we can get back the binary one by defining that A and C are independent iff there is some evidence E such that A and C are independent on evidence E. Gärdenfors (1978, 1990) considers ternary dependence relations. But there seems to be only very little interaction between the evidence E on the one hand and A and C on the other, in the sense that there are no interesting properties linking them. Therefore, in order to improve readability we have restricted our analysis to binary dependence relations here, but it is clear that a ternary relation is certainly the most general one. Our analysis should carry over to the ternary case with only little modifications.

8 Acknowledgements

This work has been supported by the project ESPRIT BRA DRUMS II and the French 'projet inter-PRC Raisonnement sur l'évolutif et l'incertain dans une base de connaissances'. The paper was partly written during a stay of the first author at the University of Konstanz that has been supported by CNRS and DAAD. He wishes to thank Hans Rott for the useful discussions he had there.

9 REFERENCES

C. E. Alchourrón, P. Gärdenfors and D. Makinson (1985), On the logic of theory change: partial meet contraction and revision functions. J. of Symbolic Logic 50, 510-530.

S. Benferhat, D. Dubois, H. Prade (1994), Expressing independence in a possibilistic framework and its application to default reasoning. Proc. ECAI-94.

R. Cohen (1994), Some steps towards a general theory of relevance. Synthese 101, pp 171-185.

J. Delgrande and F. J. Pelletier (1994), A formal approach to relevance: extended abstract. Proc. AAAI Fall Symposium, AAAI Press.

D. Dubois, H. Prade (1991), Epistemic entrenchment and possibilistic logic. AI Journal 50, pp 223-239.

D. Dubois, H. Prade (1992), Belief change and possibility theory. In: Belief Revision, ed. P. Gärdenfors, Cambridge Tracts in Theoretical Computer Sc. 29, Cambridge University Press.

D. Dubois, L. Fariñas del Cerro, A. Herzig, H. Prade (1994), An ordinal view of independence with application to plausible reasoning. Proc. Int. Conf. on Uncertainty in AI (UAI-94).

L. Fariñas del Cerro, A. Herzig (1994a), Possibility theory and independence. In: B. Bouchon-Meunier, R.R. Yager, L. A. Zadeh, eds., Advances in Intelligent Computing - IPMU-94. Springer Verlag, LNCS 945, pp 292-301.

L. Fariñas del Cerro and A. Herzig (1994b), Interference logic = conditional logic + frame axiom. Int. J. of Intelligent Systems, Special issue on Revision and Updating in Knowledge Bases, 9(1), pp 119-130.

L. Fariñas del Cerro, A. Herzig (1994c), A conditional logic for updating in the Possible Models Approach. Proc. 18th German Conf. on Artificial Intelligence (KI-94), eds. B. Nebel, L. Dreschler-Fischer, Springer Verlag, LNAI 861.

L. Fariñas del Cerro, A. Herzig (1996), Revisions, updates, and interference. In: Logic, Action, and Information, eds. A. Fuhrmann, H. Rott, DeGruyter (Berlin-New York), pp 189-207.

P. Gärdenfors (1978), On the logic of relevance. Synthese 37, pp 351-367.

P. Gärdenfors (1990), Belief revision and irrelevance. PSA, Vol. 2, pp 349-356.

O. Gasquet and A. Herzig (1995), Reasoning about actions using dependence. Internal Report, IRIT, jan. 1995.

V. Lugardon (1996), Sur les fondements de la notion de dépendance et de son application à la théorie de l'action. PhD thesis, Université Paul Sabatier, Toulouse, jan. 1996.

L. Fariñas del Cerro, V. Lugardon (1991), Sequents for dependence logics. Logique et Analyse 133-134, pp 57-71.

D. Makinson, P. Gärdenfors (1990), Relations between the logic of theory change and nonmonotonic logic. In: The Logic of Theory Change, eds. A. Fuhrmann, M. Morreau, Springer Verlag, LNCS 465, pp 185-205.

J. Pearl (1988), Probabilistic reasoning in intelligent systems: Networks of plausible inference. Morgan Kaufmann.

W. Spohn (1994), On the properties of conditional independence. In: P. Humphreys (ed.), Patrick Suppes: Scientific philosopher, Vol. 1, Kluwer Academic Publishers.

M. Studený (1993), Formal properties of conditional independence in different calculi of AI. Proc. of the Eur. Conf. on Symbolic and Quantitative Approaches to Reasoning and Uncertainty (ECSQARU'93), Springer Verlag, LNCS 747, 341-348.