Nonmonotonic default modal logics
(Detailed abstract)

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ABSTRACT. Conclusions by failure to prove the opposite are frequently used in reasoning about an incompletely specified world. This naturally leads to logics for default reasoning which, in general, are nonmonotonic, i.e., introducing new facts can invalidate previously made conclusions. Accordingly, a nonmonotonic theory is called (nonmonotonically) *degenerate*, if adding new axioms does not invalidate already proved theorems. We study nonmonotonic logics based on various sets of defaults and present a necessary and sufficient condition for a nonmonotonic modal theory to be degenerate. In particular, this condition provides several alternative descriptions of degenerate theories. Also we establish some closure properties of sets of defaults defining a nonmonotonic modal logic.

1. Introduction

Nonmonotonic reasoning is very natural in Artificial Intelligence. For example, when an expert system derives a conclusion based on incomplete knowledge, this conclusion may be invalidated in the future by the new facts about the external world. In Prolog, with its *negation by failure* semantics, the proved goals can become invalid after the addition of new facts to the data base. Also while dealing with probabilistic reasoning, the derived probabilities of different events can change completely, when new facts are added to the knowledge base. Thus if one uses threshold probabilities for making conclusions, the accepted truths may change as well.

Logics which reflect nonmonotonic reasoning have been first introduced in [2], [7], [8], and [12]. More general approach to the question "What is a nonmonotonic system?" can be found in [1], [3], [5], and [6]. In particular, a detailed example of nonmonotonic reasoning can be found in [2]. Most of nonmonotonic logics are based on semantics and proof theory, both obtained via fixed points of some monotonic operators. The *default logic* of Reiter ([12]) is based on theories which are fixed points of such an operator. The logic of McDermott and Doyle ([8]) is based on the intersection of all fixed points of a similar operator. The *circumscription* of McCarthy ([7]) is based on a definition of a predicate as the minimal relation satisfying some property.

Later, McDermott in [9] introduced nonmonotonic modal logics which are based on the modal systems T, S4, and S5. These modal logics are more suitable for describing dynamic worlds. However, his logics are a little bit problematic in view of the following. First, it is unknown whether McDermott's logics based on the first order versions of T and S4 are consistent. In addition, the logic based on S5 degenerates to the monotonic one, cf. [9].

We shall study here nonmonotonic modal logics, which contain a *possibility operator M* and a *necessity operator L*. More precisely, logics which are extensions of the modal system T. Our definition of nonmonotonics logics is a relativization of that appearing in [9], namely, the nonmonotonic theory is the intersection of all extensions of the default theory presented in ([12]). The main difficulty of dealing with a nonmonotonic modal logic is that the underlying monotonic modal logic lacks a deduction theorem ($A, \varphi \vdash \psi$ implies $A \vdash \varphi \supset \psi$). For this reason we cannot prove that every (monotonically) consistent theory has a consistent nonmonotonic fixed point, etc., cf. [9] and Proposition 4 in Section 2. Despite of this, in modal logics which are extensions of T we have a "weak deduction theorem" stating that $A, \varphi \vdash \psi$ implies $A \vdash L^k\varphi \supset \psi$, for some k, where $L^0\varphi$ is φ, and $L^{k+1}\varphi$ is $LL^k\varphi$. Using this weak deduction theorem we can give a condition for a nonmonotonic default logic to be *degenerate*, i.e., to become monotonic. This condition is the main result of this paper, and states that a default modal logic degenerates if and only if the set of defaults is, in some sense, closed under negation. In particular, it provides an alternative proof of the degeneration of McDermott's S5.

Another version of nonmonotonic modal logics discussed in literature is *autoepistemic logic*, cf. [10], [11], and [4]. This logic is based on the modal logic K45 and restricted to the application of default reasoning to nonmodal formulas. Since T is not a sublogic of K45, the theory developed in this paper is not applicable to autoepistemic logics. However, it is possible to find some similarity between autoepistemic logic and the nonmonotonic *ground* logics introduced in Section 4.

The paper is organized as follows. In the next section we give the necessary definitions and derive some simple properties of nonmonotonic default logics. Section 3 contains the main result of this paper, i.e., a condition for the degeneration of nonmonotonic modal logics. Also in that section we prove that for any nonmonotonic default logic the set of defaults can be taken to be closed under the operators $\wedge$, $\vee$, and L. In Section 4, we present a slightly different version of McDermott's nonmonotonic logic that both is consistent and nondegenerate.

2. Monotonic and nonmonotonic modal logics

This section is organized as follows. First we give definitions of monotonic and nonmonotonic modal logics and derive some of their properties. Next, we discuss the nonmonotonic modal logics of McDermott ([9]), which constitute a particular case of nonmonotonic default modal logics.

The language **Lang** of modal logic is obtained from the language of the (first order) predicate calculus by extending it with a modal connective L (*necessarily*). As usual, the dual connective M (*possibly*) is defined by $\sim L \sim$. A formula without free variables is called a *sentence*, and the set of all sentences is denoted by **St**. We assume that **Lang** is countable.

In this paper we shall deal with modal logics which result from the classic predicate calculus by adding the rule of inference

Necessitation (NEC): $\varphi \vdash L\varphi$,

and all the instances of some subsets of the axiom schemata below.

M1. $L\varphi \supset \varphi$

M2. $L(\varphi \supset \psi) \supset (L\varphi \supset L\psi)$

M3. $\forall x\, L\varphi \supset L\forall x \varphi$

M4. $L\varphi \supset LL\varphi$

M5. $M\varphi \supset LM\varphi$

The system T contains axiom schemata M1 and M2 only. Adding M4 to T results in S4, and adding M5 to S4 results in S5. In this paper by modal logic we refer to any modal system that is an extension of T + M3 with additional axioms, e.g., T + M3 itself, S4 + M3, S5 + M3, etc.. Below these systems will be simply denoted by T, S4, and S5, respectively.

For a set of formulas $A \subseteq$ **Lang**, called *axioms*, we define the (monotonic) *theory of* A, denoted by Th(A), as

$$\mathrm{Th}(A) = \{\, \varphi \in \mathbf{Lang} : A \vdash \varphi \,\} \subseteq \mathbf{Lang}.$$

As usual, we write $A \vdash \varphi$, if there exists a sequence of formulas $\psi_1, \psi_2, \ldots, \psi_n = \varphi$ such that each ψ_i is an axiom or belongs to A or is obtained from some of the formulas $\psi_1, \psi_2, \ldots, \psi_{i-1}$ by one of the rules of inference: *modus ponens, generalization* or *necessitation*. Thus the relation $\vdash$ and the operator Th should be subscripted by T, S4 or S5, respectively. However, in this paper, if not specified otherwise, the results are true for every modal logic containing T, and the subscripts will be omitted.

Let $D \subseteq \mathbf{St}$ be a set of sentences called *defaults*. Following [8], [9], and [12] we define a *default logic* by adding to a modal logic, roughly speaking, the following "rule of inference".

$$\frac{\nvdash \sim\varphi}{\varphi}, \quad \varphi \in D. \tag{1}$$

This rule is read as

"for a default $\varphi \in D$, derive φ if $\sim\varphi$ is not provable".

However, the above rule is self-referring, and therefore it is ill-defined. A possible correct definition of nonmonotonic inference is given below. It is similar to that appearing in [9].

Definition 1. The nonmonotonic modal D-default theory of $A \subseteq \mathbf{Lang}$, denoted by $\mathbf{NTH}_D(A)$, is the intersection of **Lang** and all the *fixed points* of the operator $\mathbf{NM}_D^A$, defined below.

For a set of formulas F, $\mathbf{NM}_D^A(F)$ is defined by

$$\mathbf{NM}_D^A(F) = \mathrm{Th}(A \cup \mathbf{As}_D^A(F)),$$

where

$$\mathbf{As}_D^A(F) = \{ \varphi \in D : \sim\varphi \notin F \} - \mathrm{Th}(A).$$

A set of formulas X is called a fixed point of $\mathbf{NM}_D^A$, if $\mathbf{NM}_D^A(X) = X$.

Thus

$$\mathbf{NTH}_D(A) = \mathbf{Lang} \cap \bigcap \{ X : X = \mathbf{NM}_D^A(X) \}.$$

Remark 1. Since $\bigcap \varnothing = \mathbf{Lang}$, we can define $\mathbf{NTH}_D(A)$ as $\bigcap \{ X : X = \mathbf{NM}_D^A(X) \}$. Also we trivially have

$$\mathbf{NM}_D^A(F) = \mathrm{Th}(A \cup \{ \varphi \in D : \sim\varphi \notin F \}),$$

because

$$\mathrm{Th}(A \cup \{ \varphi \in D : \sim\varphi \notin F \}) = \mathrm{Th}(A \cup (\{ \varphi \in D : \sim\varphi \notin F \} - \mathrm{Th}(A))) = \mathrm{Th}(A \cup \mathbf{As}_D^A(F)).$$

Similarly to [12], a fixed point of $\mathbf{NM}_D^A$ can be considered as an acceptable set of beliefs that one may hold about incompletely specified changing world. I.e., a fixed point of $\mathbf{NM}_D^A$ realizes some defaults and rejects all the others. Alternatively, such a fixed point can be thought of as a "syntactic model" for A, or as a "minimal complete for D extension" of A with formulas from D. The nonmonotonic theory of A is the set of formulas which are believed in all the fixed points.

A set of axioms A is said to be *nonmonotonically inconsistent* (with respect to D), if $\mathbf{NTH}_D(A) = \mathbf{Lang}$, i.e., each formula can be derived in an inconsistent theory, exactly as in the case of monotonic logics. In particular, if for the set of axioms A, $\mathbf{NM}_D^A$ has no fixed points (models), then A is (nonmonotonically) inconsistent, because in this case we have $\mathbf{NTH}_D(A) = \mathbf{Lang}$. At the end of this section we present an example of a (monotonically) consistent set of axioms whose induced operator has no fixed points with respect to some set of defaults. This example is related to the nonmonotonic modal logics of McDermott, and to the nonmonotonic ground logics introduced in Section 4.

Another possibility for a set of axioms A to be (nonmonotonically) inconsistent is indicated by Proposition 1 below.

Proposition 1. ([12]) **Lang** *is a fixed point of* $\mathbf{NM}_D^A$ *if and only A is (monotonically) inconsistent. In this case* **Lang** *is the only fixed point of* $\mathbf{NM}_D^A$.

In this paper, if not specified otherwise, the words "consistent" and "inconsistent" refer to the monotonic case.

Fixed points of $\mathbf{NM}_D^A$ can be alternatively described by the following proposition.

Proposition 2. *Let F be a proper subset of* **Lang**. *Then F is a fixed point of* $\mathbf{NM}_D^A$ *if and only if it satisfies the following two conditions.*

(i) $F = \mathrm{Th}(A \cup (F \cap D))$, *and*

(ii) For any $\varphi \in D$ *either* $F \vdash \varphi$ *or* $F \vdash \sim\varphi$, *i.e., "F is complete for D".*

Condition (*i*) states that a fixed point is generated by the formulas added by the rule of nonmonotonic inference, i.e., that this rule is the only one used. Condition (*ii*) states that the rule of nonmonotonic inference is satisfied.

Corollary 1. ([8], [12]) *Let* F_1 *and* F_2 *be fixed points of* $\mathbf{NM}_D^A$. *If* $F_1 \subseteq F_2$, *then* $F_1 = F_2$.

Corollary 2. ([12]) *Let* $D' \subseteq D$ *be a set of defaults. Then any consistent fixed point of* $\mathbf{NM}_D^{A \cup D'}$ *is also a fixed point of* $\mathbf{NM}_D^A$.

Remark 2. Proposition 2 implies that if the set of defaults D is of finite cardinality n, then any set of axioms has at most 2^n fixed points. Therefore, in the propositional nonmonotonic modal logic based on a finite set of defaults, if the set of axioms A is finite, then the nonmonotonic theory $\mathbf{NTH}_D(A)$ is decidable. The decision procedure is as follows. Using condition *(ii)* of Proposition 2 and the decidability of propositional modal logics T, S4, and S5, it is possible to find all subsets D' of D such that $\mathrm{Th}(A \cup D')$ is a fixed point of $\mathbf{NM}_D^A$, i.e., satisfies condition *(ii)* of Proposition 2. Then for a formula φ one can decide whether for every D' as above we have $A \cup D' \vdash \varphi$, i.e., whether φ belongs to all fixed points of A.

Nonmonotonic default logics can be illustrated by the following example. In [9] McDermott introduced the nonmonotonic modal theory of A, denoted by $\mathrm{TH}(A)$, that is the intersection of **Lang** and all the fixed points of the operator NM_A.

NM_A is defined by

$$\mathrm{NM}_A(F) = \mathrm{Th}(A \cup \mathrm{As}_A(F)),$$

where

$$\mathrm{As}_A(F) = \{ M\varphi : \varphi \in \mathbf{St},\ \sim\varphi \notin F \} - \mathrm{Th}(A).$$

Thus

$$\mathrm{TH}(A) = \mathbf{Lang} \cap \bigcap \{ X : X = \mathrm{NM}_A(X) \}.$$

The above logic reflects the following "rule of inference" called *possibilitation*.

$$\frac{\nvdash \sim\varphi}{M\varphi}.$$

By the following proposition, this rule is equivalent to default rule (1) with the set of defaults $D_M = \{ M\varphi : \varphi \in \mathbf{St} \}$.

Proposition 3. *We have* $\mathrm{TH}(A) = \mathbf{NTH}_{D_M}(A)$, *where* $D_M = \{ M\varphi : \varphi \in \mathbf{St} \}$.

It was shown in [9] that McDermott's nonmonotonic based on S5 is equivalent to the (monotonic) S5 itself. Thus, trivially, it is consistent, i.e., the empty set of axioms is nonmonotonically consistent. Also, even though McDermott's nonmonotonic logics based on the propositional versions of T and S4 are consistent, cf. [9], nothing is known about the consistency of nonmonotonic logics based on the first order versions of T and S4. However it is not hard to show that the first order nonmonotonic T and S4 with *strong equality*, i.e., $M(x=y) \supset L(x=y)$, are consistent.

In Section 4 we present a slightly modified version of McDermott's logic, called nonmonotonic ground logic. This logic is (nonmonotonically) consistent and possesses many of the "nonmonotonic" properties of McDermott's logic. Moreover, it is nondegenerate even when the underlying modal logic is S5.

We close this section by an example of a consistent set of axioms that has no fixed points.

Proposition 4. *Let the underlying modal logic be first order T or S4, and let ψ be a sentence not containing modal connectives such that $\nvdash \psi$. If the set of defaults D is a subset of $D_M = \{ M\varphi : \varphi \in \mathbf{St} \}$ and contains $M{\sim}\psi$, then the set of axioms $\{ML\psi\}$ is (nonmonotonically) inconsistent.*

3. Closure properties of sets of defaults and degeneration of nonmonotonic theories

First we establish a closure property of the set of defaults under the positive connectives $\wedge$, $\vee$, and L. This closure property can be considered as a motivation for Theorems 2 and 3 below.

Definition 2. Let $D \subseteq \mathbf{Lang}$. We say that D is *closed* under connectives $\wedge, \vee$, and L, if $\varphi, \psi \in D$ implies $\varphi \wedge \psi, \varphi \vee \psi, L\varphi \in D$. We define $\bar{D}$, the *closure* of D under the connectives $\wedge$, $\vee$, and L, to be the set of all formulas which can be obtained from formulas of D by means of the connectives $\wedge$, $\vee$, and L.

Theorem 1. *For every set of defaults D we have $\mathbf{NTH}_D(A) = \mathbf{NTH}_{\bar{D}}(A)$. Moreover, $\mathbf{NM}_D^A$ and $\mathbf{NM}_{\bar{D}}^A$ have the same fixed points.*

Theorem 1, naturally, suggests to ask what about the closure under negation. But as is shown in the sequel, if the set of defaults is closed under negation, then the corresponding nonmonotonic logic is monotonic.

Next we present the main result of the paper, namely, a condition for a nonmonotonic modal logic to degenerate to a monotonic one. In order to give a precise statement of this condition we observe that for any default $\varphi \in D$ and any fixed point F we have $L^i\varphi \vee L^j{\sim}\varphi \in F$, $i, j = 0, 1, \ldots$. Indeed, if $F = \mathbf{Lang}$, then the proposition is, trivially, true. Otherwise, by Proposition 2, either $F \vdash \varphi$, or $F \vdash {\sim}\varphi$. In the former case, by i applications of NEC, $F \vdash L^i\varphi$, which, in turn, implies $L^i\varphi \vee L^j{\sim}\varphi \in F$, because F is deductively closed. The case of $F \vdash {\sim}\varphi$ is treated similarly.

The set of formulas $\{L^i\varphi \vee L^j{\sim}\varphi : \varphi \in D,\ i, j = 0, 1, \dots\}$ will be referred to as the set of *axioms imposed by D* and will be denoted by Ax_D. In this notation the above observation can be restated as follows.

Proposition 5. *We have* $\mathrm{Th}(A \cup \mathrm{Ax}_D) \subseteq \mathbf{NTH}_D(A)$.

Now consider the properties of nonmonotonic default modal logics stated below.

1. For every default $\varphi \in D$ there exists a default $\psi \in D$ such that $A, \psi \vdash {\sim}\varphi$, and $A, \mathrm{Ax}_D, {\sim}\varphi \vdash \psi$. This property of D can be thought as "the closure under negation relatively to A".

2. $\mathbf{NTH}_D(A) = \mathrm{Th}(A \cup \mathrm{Ax}_D)$, i.e. the nonmonotonic theory on A is equal to the monotonic one augmented with the additional axioms imposed by D. Notice that by Proposition 6, Ax_D is the least set of additional axioms that could enjoy this property.

2′. For every $A' \supseteq A$, $\mathbf{NTH}_D(A') = \mathrm{Th}(A' \cup \mathrm{Ax}_D)$, i.e. the nonmonotonic theory on extensions of A is equal to the monotonic one augmented with the additional axioms imposed by D.

3. For every $A' \supseteq A$, $\mathbf{NTH}_D(A') \supseteq \mathbf{NTH}_D(A)$, i.e. the nonmonotonic theories of extensions of A do not invalidate the assumptions (nonmonotonically) deduced from A. In other words, the operator $\mathbf{NTH}_D$ is monotonic in A.

3′. For every $A'' \supseteq A' \supseteq A$, $\mathbf{NTH}_D(A'') \supseteq \mathbf{NTH}_D(A')$, i.e., the logic is monotonic in the extensions of A.

Theorems 2 and 3 below show that the above properties of nonmonotonic theories are tightly connected.

Theorem 2. *For any set of defaults D and for any set of axioms A we have*

$$\begin{array}{ccccc} 1 & \Rightarrow & 2' & \Rightarrow & 2 \\ & & \Updownarrow & & \Updownarrow \\ & & 3' & \Rightarrow & 3\,, \end{array}$$

where $\Leftrightarrow$ *denotes equivalence, and* $\Rightarrow$ *denotes implication.*

In order to close the diagram given by Theorem 2 we need additional assumptions on the set of defaults and the underlying modal logic.

Definition 3. We shall say that a set of formulas $\Phi \subseteq \mathbf{Lang}$ is *finitely based* if there exist formulas $\varphi_1, \varphi_2, \dots, \varphi_n$ such that every formula of Φ can be obtained from $\varphi_1, \varphi_2, \dots, \varphi_n$ by means of propositional and modal connectives. I.e., for every formula $\varphi \in \Phi$ there exits a formula φ' in the language of the propositional modal logic over the propositional variables $p_1, p_2, \dots, p_n$ such that φ results by the substitution of φ_i for p_i in φ', $i = 1, 2, \dots, n$. The set of formulas $\{\varphi_1, \varphi_2, \ldots, \varphi_n\}$ is called *a finite base for* Φ.

Theorem 3. *If the underlying modal logic contains* S4 *and the set of defaults is finitely based and closed under* $\wedge$, $\vee$, *and* L, *then property* **2** *implies property* **1**, *i.e., the five properties stated above are equivalent.*

In Theorem 3, the condition imposed on the set of defaults to be closed under $\wedge$, $\vee$, and L is required only for a technical reason. (Alternatively, in view of Theorem 1, we could talk about $\tilde{D}$ in property **1**.) However, it can be shown that the requirement of a finite base is essential.

Next we present some of almost immediate corollaries to Theorems 2 and 3. The first one gives a proof-theoretic version of the corresponding result in [9].

Corollary 1. *Let* TH(A) *be the nonmonotonic theory of McDermott defined in Section 2. Then* TH(A) = Th(A) *if and only if* Th(A) *contains the sentential part of* M5.

Corollary 2. *Let* **Lang** *be a language of propositional modal logic of finite signature (that is the set of propositional variables is finite), and let the underlying modal logic contain the propositional part of* S4. *If the set of defaults D is closed under* $\wedge$, $\vee$, *and L, then all the properties* **1**, **2**, **2′**, **3**, *and* **3′** *are equivalent.*

Corollary 3. *Let the set of defaults D be finitely based and let the underlying modal logic contain* S4. *If* $\mathrm{Th}(\varnothing) = \mathbf{NTH}_D(\varnothing)$, *then* $\mathrm{Th}(A) = \mathbf{NTH}_D(A)$ *for every set of axioms A.*

Remark 3. It can be easily shown that $L\varphi \vee L{\sim}\varphi \vdash_{\mathrm{T}} L^i\varphi \vee L^j{\sim}\varphi$, $i, j = 0, 1, \ldots$. Thus Ax_D could be defined as $\{L\varphi \vee L{\sim}\varphi\}_{\varphi \in D}$.

4. Nonmonotonic ground logics

One of the undesirable properties of the nonmonotonic logics of McDermott is that a consistent set of axioms may have no fixed points, i.e., be nonmonotonically inconsistent. A possible reason for this may be the lack of clear separation between the defaults not containing modalities, which one can consider as the facts about the real world, and the defaults containing modalities, which are "metaformulas" supposed to interpret knowledge, necessity, contingency, etc.. In this section we propose a slightly modified version of the nonmonotonic logics of McDermott that seems to be more convenient to deal with. These logics, referred to as *nonmonotonic ground logics*, result from the set of defaults D_G that is defined as follows.

$$D_G = \{ M\varphi : \varphi \text{ is a sentence without modalities} \}.$$

In view of [4, Proposition 3.6], fixed points of nonmonotonic ground logics correspond to *minimal* autoepistemic extensions. However, the language of nonmonotonic ground logics is richer than that of the autoepistemic one, because the language of autoepistemic logic does not allow the occurrence of modal operators within the scope of quantifaers. In addition an S5-consistent set of axioms is also nonmonotonically consistent, cf. Proposition 8 below, whereas in autoepistemic logics there exist consistent sets of formulas which have no autoepistemic extension, cf. [4, Example 2.2].

As in the case of McDermott's logics based on T or S4, the consistent set of axioms $\{MLp\}$ is inconsistent in nonmonotonic ground logic based on T or S4, cf. Proposition 4. But, fortunately, for T, S4 and S5 every consistent set of axioms without modalities is also nonmonotonically consistent. Moreover, for S5 this is true for any set of axioms, even if it contains "metaformulas". The former, in particular, implies that the first order nonmonotonic theory resulting from the empty set of axioms is consistent in nonmonotonic ground logic, even if the underlying modal logic is T or S4. In addition, nonmonotonic ground logic is nondegenerate in S5. The precise statements of the above results are given below.

Proposition 6. *Let the underlying logic contain T and be contained in S5, and let A be a consistent set of axioms without modalities. Then* $\mathbf{NM}^A_{D_G}$ *has a unique consistent fixed point* $F_A = \mathrm{Th}(A \cup \{ M\varphi \in D_G : A \nvdash \sim\varphi \})$.

Proposition 7. *Let the underlying logic contain T and be contained in S5. If a set of axioms A does not contain modalities, and* $\mathrm{Th}(A)$ *is not complete in the predicate calculus, then there exists a consistent set of axioms* $A' \supset A$ *without modalities, such that* $\mathbf{NTH}_{D_G}(A') \not\supseteq \mathbf{NTH}_{D_G}(A)$.

Proposition 8. *Let the underlying modal logic be* S5. *If a set of axioms A is consistent, then* $\mathbf{NTH}_{D_G}(A)$ *is also consistent.*

Finally we would like to note that, in view of Remark 2 in Section 2, nonmonotonic propositional ground logics over a finite signature are decidable, because their set of defaults D_G is finite.

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